Insurance Demand and the Mitigation of Default Risk

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Mossin's Demand Model as Starting Point

Mossin, J. (1968). Aspects of Rational Insurance Purchasing. Journal of Political Economy, 76, 533-568 → *Full coverage is optimal iff premium is actuarially fair*

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Mossin (1968) (default-free setting)

$$W := w - c_I(\alpha_I) + (-1 + \alpha_I)L,$$

$$L = \begin{cases} l, & if\ loss\ occurs, prob\ p \\ 0, & else, prob\ 1-p \end{cases}$$

Modified Demand Model: The Case of Non-Performance

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Doherty N., Schlesinger H. (1990). Rational Insurance Purchasing: Consideration of Contract Nonperformance. Quarterly Journal of Economics, 105, 243-253 → *Given default risk, over- or under-insurance may be optimal even though the premium is actuarially fair*

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$$W := w - c_I(\alpha_I) + \{-1 + \alpha_I\boldsymbol{(1 - \chi_D(1 - r))}\}L, \quad \chi_D = \begin{cases} 1, & if\ insurer\ fails, prob\ q \\ 0, & else, prob\ 1-q \end{cases}$$

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↳ **Mahul O., Wright B. D. (2007)**. Optimal Coverage for Incompletely Reliable Insurance, Economic Letters 95, 456-461 → *Optimality of under- or over-insurance depends on size of recovery rate* r

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↳ Optimal insurance demand under the risk of contract nonperformance, if default risk can be diversified?

→ *multiple co-insurance as diversification measure*

Our Model Framework

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Our model framework

n co-insurers: each co-insurer holds $\frac{1}{n}$ in premium and losses

$$W_{F,n} := w - c_I(\alpha_I, \boldsymbol{n}) + \left\{-1 + \alpha_I\left(1 - \frac{\boldsymbol{F}}{\boldsymbol{n}}(1 - r)\right)\right\}L,$$

α_I is our decision variable: coverage rate of co-insurance policy

- $F \in \{0, \dots, n\}$ is the random number of failed insurers
- $F \sim BB\left(q\frac{1-\theta}{\theta}, (1-q)\frac{1-\theta}{\theta}\right)$, θ is the joint default correlation factor
- As $\theta \to 0$, F approaches a binomial distribution (independent defaults)

Remarks on The Model: Probability Weights, Utility and Premium Principle

Policyholder's utility (is maximized with respect to α_I)

$$U = (1-p)u(W_{no\ loss}) + p\sum_{k=0}^{n} d_{k,n}u\left(W_{k,n}\right)$$

Probability of *k* failing co-insurers (given a policy with *n* co-insurers)

$$d_{k,n} = \mathbb{P}[F=k] = \int_0^1 \binom{n}{k} x^k\,(1-x)^{n-k}\psi(x;\alpha,\beta)\;dx$$

($\psi(x;\alpha,\beta)$ is the density of a Beta distribution with parameters $\alpha = q\frac{1-\theta}{\theta}$ and $\beta = (1-q)\frac{1-\theta}{\theta}$)

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Assumed premium principle: *Expected Payoff* x *Proportional Cost Loading*

$$\begin{aligned} c_I(\alpha_I, n) &= E[Default\ Adjusted\ Indemnification](1+\lambda_I) \\ &= \alpha_I Lp\{1-q(1-r)\}(1+\lambda_I) = c_I(\alpha_I) \end{aligned}$$

Independent of n and θ!

Each insurer receives $\frac{1}{n} c_I(\alpha_I) \to 0$, as n becomes large: fixed running costs?

Effect of Diversification on Optimal Demand

- Assumed the insurer can increase the number of co-insurers from n to n+1 → Natural question: Is it optimal to **increase** or to **decrease** insurance **coverage**?
- First intuition: Given two policies, it seems to be nearby that it is optimal to take up more of the policy that provides higher utility.

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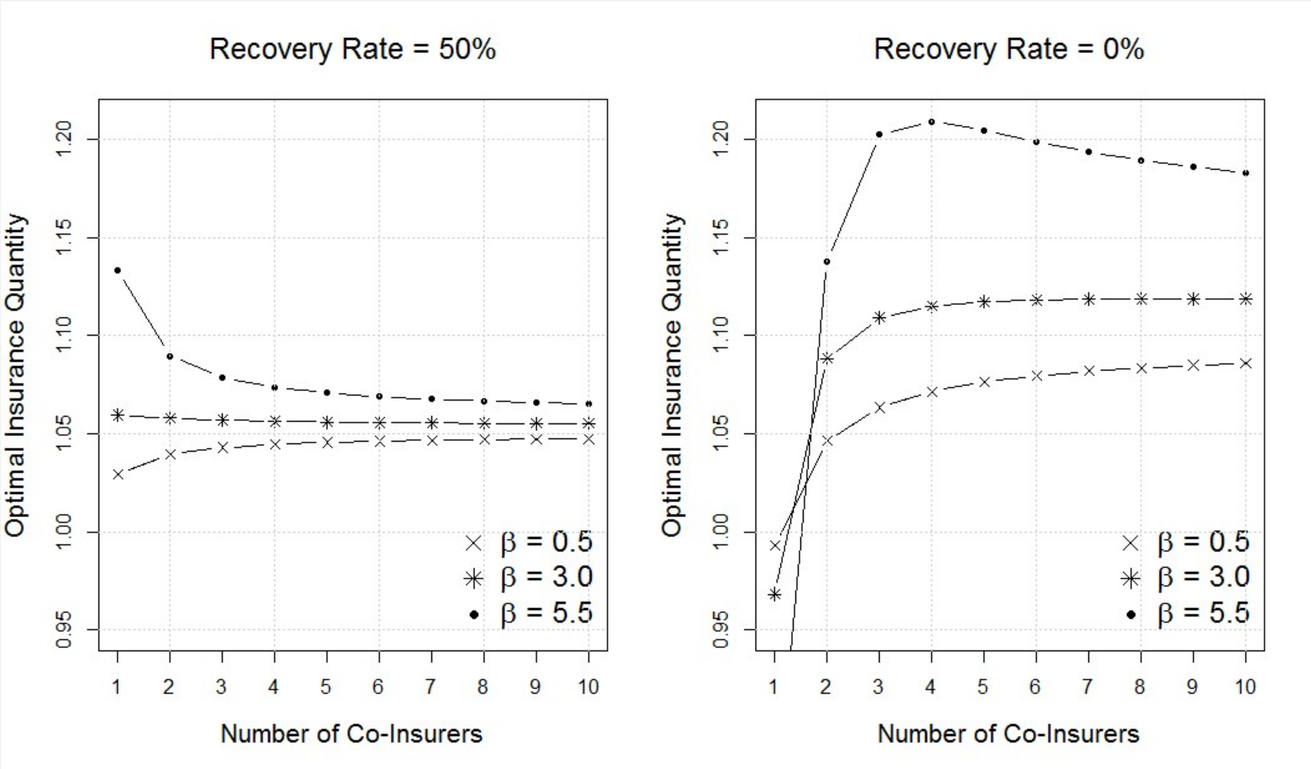

Numeric Example:

$$u(x) = -exp(-\beta x)$$

Initial wealth w	1.5
Loss prob. p	5.0 %
Loss size l	1.0
Default prob. q	1.0 %
Correlation θ	15 %
Cost loading λ_I	0.0

Monotonicity Criterion

Let $\alpha_{I,n}^*$ *be the optimal insurance demand for* n *co-insurers and set*

$$w_n^*(x) = w - l - c\left(\alpha_{I,n}^*\right) + \alpha_{I,n}^* l - \alpha_{I,n}^*(1-r)lx.$$

Then, $\alpha_{I,n+1}^* \geq (\leq) \alpha_{I,n}^*$ *holds true, if*

$$\left(\mathbf{1} - \frac{\boldsymbol{w}-\boldsymbol{l}}{\boldsymbol{w}_{\boldsymbol{n}}^*(\boldsymbol{x})}\right)\boldsymbol{\eta}\left(\boldsymbol{w}_{\boldsymbol{n}}^*(\boldsymbol{x})\right) \leq (\geq)\mathbf{2}, \textbf{\textit{ for all }} \boldsymbol{x} \in [\mathbf{0},\mathbf{1}],$$

where $\eta(x) := -xu'''(x)/u''(x)$ *is the policyholder's measure of relative prudence.*

Possible cases (heuristically)

(1) Low degree of prudence → " $\leq$ " is fulfilled → optimal coverage **non-decreasing**

(2) High degree of prudence

& high recovery rate → " $\geq$ " is fulfilled → optimal coverage **non-increasing**

& low recovery rate → neither " $\geq$ “ nor " $\leq$ “ is fulfilled → unambiguous monoton.

Monotonicity Criterion: Influence of Prudence

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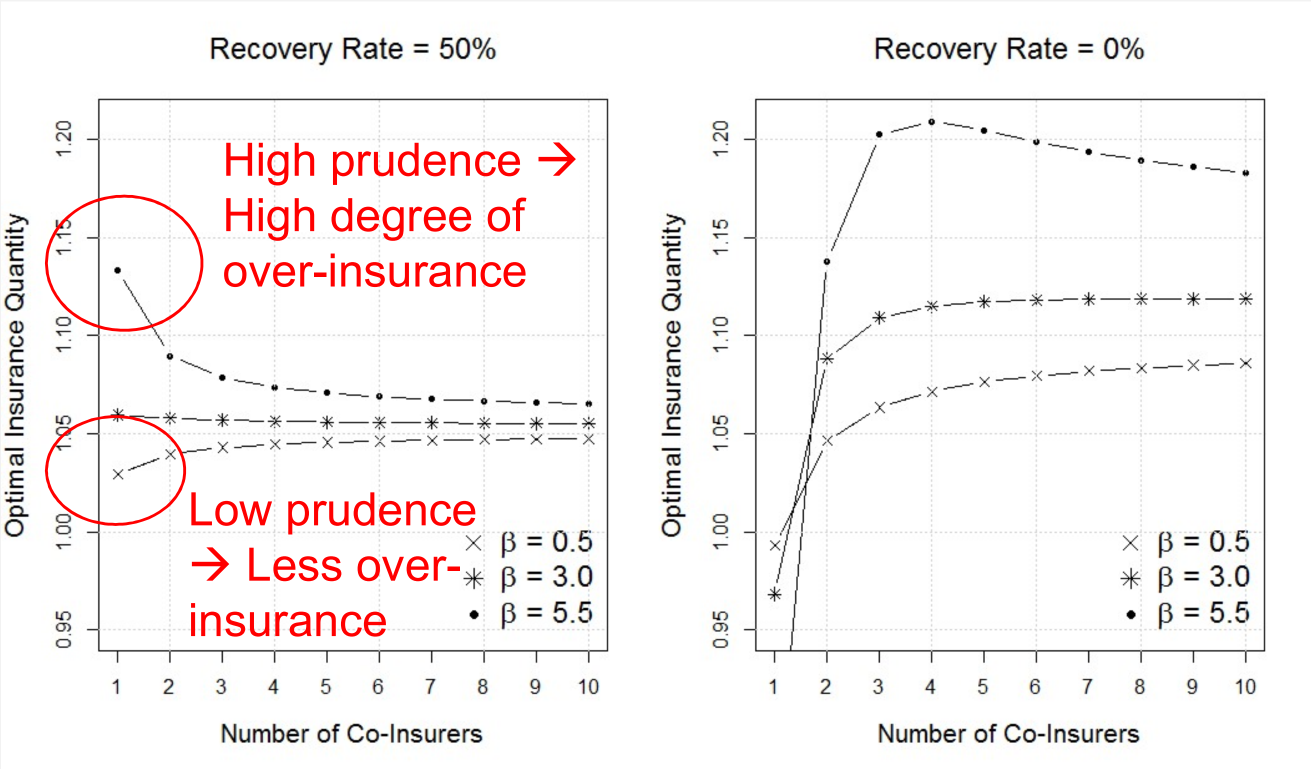

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Further Results

(1) Analogous result for default correlation: Let $\boldsymbol{\theta_1 \leq \theta_2}$

$$\left(1-\frac{w-l}{w_n^*(x)}\right)\eta\left(w_n^*(x)\right) \leq (\geq) 2,\ \textit{for all}\ x \in [0,1], \Longrightarrow \alpha_{I,n}^*(\theta_1) \geq (\leq) \alpha_{I,n}^*(\theta_2)$$

A rising default correlation thus (rather) results in a

- decreased coverage, when policyholder is low-prudent
- increased coverage, when policyholder is high-prudent and recovery rate is high

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(2) Implication for the single-insurer policy

$$\left(1-\frac{w-l}{w_1^*(x)}\right)\eta\left(w_1^*(x)\right) \leq (\geq) 2,\ \textit{for all}\ x \in [0,1], \Longrightarrow \alpha_{I,1}^* \leq (\geq) \frac{\textit{Optimal demand in a default-f}\quad \textit{setting}}{1-q(1-r)}$$

- High prudence → (Rather) less coverage than “benchmark”
- Low prudence and high recovery rate → (Rather) more coverage than “benchmark”

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Thank You

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