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A Principal Component Model for Forecasting Age- and Sex-Specific Survival Probabilities in Western Europe until the Year 2070

Patrizio Vanella, M.Sc.
Leibniz Universität Hannover
Center for Risk and Insurance
Demographic and Insurance Research Center
pv@ivbl.uni-hannover.de

Vladimir Canudas-Romo, PhD
Syddansk Universitet
Max-Planck Odense Center on the Biodemography of Aging
vcanudas@health.sdu.dk

Marius Pascariu, M.Sc.
Syddansk Universitet
Max-Planck Odense Center on the Biodemography of Aging
mpascariu@health.sdu.dk

STRUCTURE

1. INTRODUCTION AND STATE OF THE ART
2. THEORETICAL BACKGROUND AND DATA
3. PRELIMINARY RESULTS
4. CONCLUSIONS, LIMITATIONS AND OUTLOOK

Motivation:

- positive trends in life expectancy
- fertility rates are still at a relatively small level of 1.5
- smaller group of people in the working age have to finance a greater group of retired persons
- long-run planning needs to take into account the long-term development of the population structure
- Forecasting of future mortality important for political and managerial decision-making

Reasons for stochastic forecasting:

- Deterministic projections commonly based on a small number of subjective scenarios
- deterministic projections do not assign individual probabilities to identified scenarios
- Stochastic forecasts can theoretically account for infinitely many scenarios and probabilistically quantify them
- preferable for planning purposes in insurance applications (e.g. financing of the statutory pension fund)

State of the Art:

- *Bell and Monsell (1991):*
  - first mortality forecast
  - forecast ASMR for 22 age groups of U.S. females of European descent basend on PCTS model consisting of the first five PC
  - AR(1) models for each PCTS → forecasts with 95%-prediction intervals (PI) for the ASMR
- *Lee and Carter (1992):*
  - simple mortality index for forecasting the ASMR of U.S. population until 2065
  - random walk model for forecasting of all logarithmized ASMR including point-wise 95%-PI.
  - still used and popular under Lee-Carter model

1. INTRODUCTION AND STATE OF THE ART

- *Hyndman and Ullah (2007):*
  - generalization of Lee-Carter model
  - forecasting of log-ASMR of male persons in France until 2050 with 80%-PI
  - interesting approach but difficult to implement in practice
- *Raftery et al. (2013):*
  - Bayesian Hierarchical Model (BHM)
  - world mortality model for 158 countries projecting the future life expectancy for each country as a random walk with drift
  - Monte Carlo simulations lead to scenario-based PI
- *Pampel (2005):*
  - forecast of gender-gap with and without smoking-effect until 2020
  - result: increasing gender-gap assuming a similar smoking behavior between men and women

Assessment of future longevity:

- data on very old people in the population very error-prone $\Rightarrow$classical statistical models are of no use for old-age mortality estimation
- 3 major hypothesis about maximum life span:
  1. upper age limit does exist
  2. probability of survival monotonically decreases and converges to zero
  3. asymptote in survival probability over zero
- Approximation of old-age mortality:
  - Gompertz model
  - Kannisto model/ Logistic model
  - Quadratic model

2. THEORETICAL BACKGROUND AND DATA

- **<u>Goal:</u>** Propose a modeling approach for forecasting age-specific-survival rates (ASSR) by age and sex for 18 European countries until 2070
- detailed time series back to 1952
- ASMR from Human Mortality Database and Eurostat-database
- Data on deaths and population for Germany from genesis
- compute ASSR for both genders and all 18 countries based on extracted data

Approximation of ASMR for ages 90+:

- econometric extrapolation using the Kannisto model for adequate estimates of old-age mortality
- Kannisto model:

$$\mu_x = \frac{a \cdot exp(bx)}{1 + a \cdot exp(bx)}$$

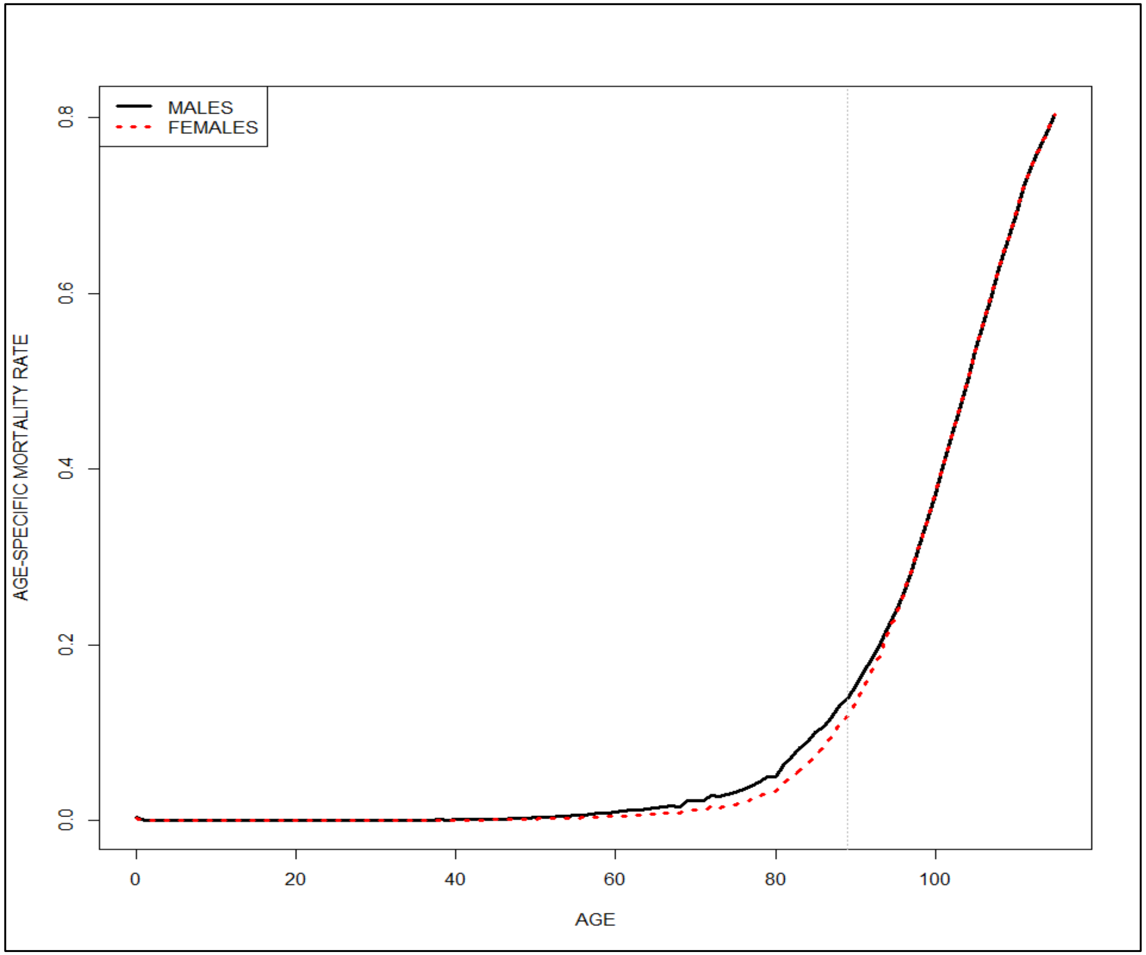
- nonlinear least squares estimation for ASMR for ages 90+:

$$m_{c,t}(x) = \frac{\alpha_{c,t} \cdot exp\left(\beta_{c,t} x\right)}{1 + \alpha_{c,t} \cdot exp\left(\beta_{c,t} x\right)}$$

- ASSR logically restricted to interval (0;1) → logistic transformation

3. PRELIMINARY RESULTS

Figure 1: Approximated old age ASMR in Germany 2014



Sources: GENESIS database; own calculation and design

Principal Component Analysis (PCA):

- PCA performed for all 18 countries simultaneously (4‘176 time series)
- General form:

$$PC_i = \sum_{j=1}^{J} e_{ij} \cdot logit(ASSR_j)$$

- Matrix notation:

$$\boldsymbol{C} = logit(\boldsymbol{S}) \times \boldsymbol{E}$$

- model first two PC in detail
- Remaining 4‘174 time series assumed Random Walks

Forecasting:

- smooth data by parametric models, fit remaining part to ARIMA models
- use resulting time series models for simulating PC until 2070 10‘000 times (Wiener‘s processes)
- Transformation

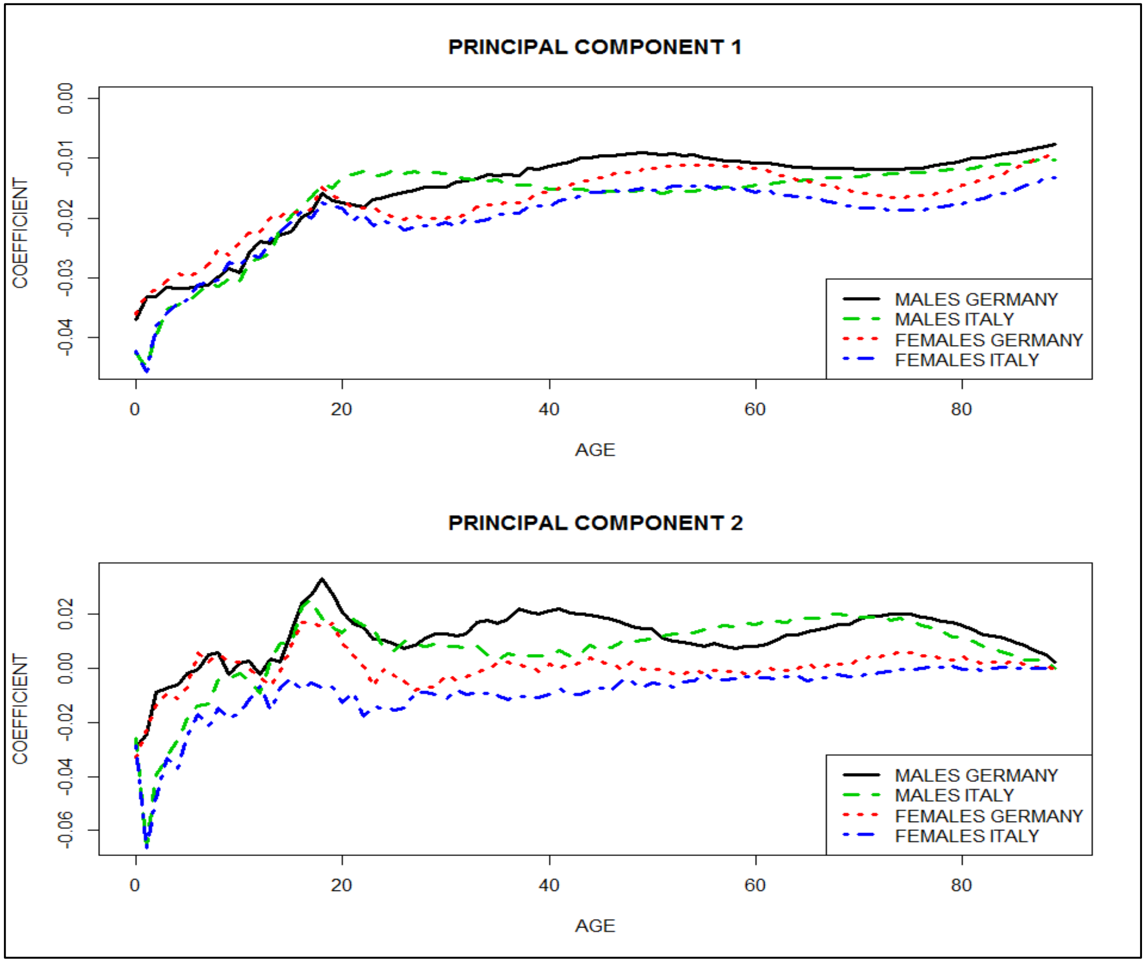
$$logit^{-1}(\hat{\boldsymbol{C}}_t \times \hat{\boldsymbol{E}}^{-1}) = \hat{\boldsymbol{S}}_t$$

  leads to simulated values for each ASSR annually

- construct pointwise PI for future value of each ASSR until 2070

3. PRELIMINARY RESULTS

Figure 2: Correlation between PC and ASSR



Source: Own calculation and design

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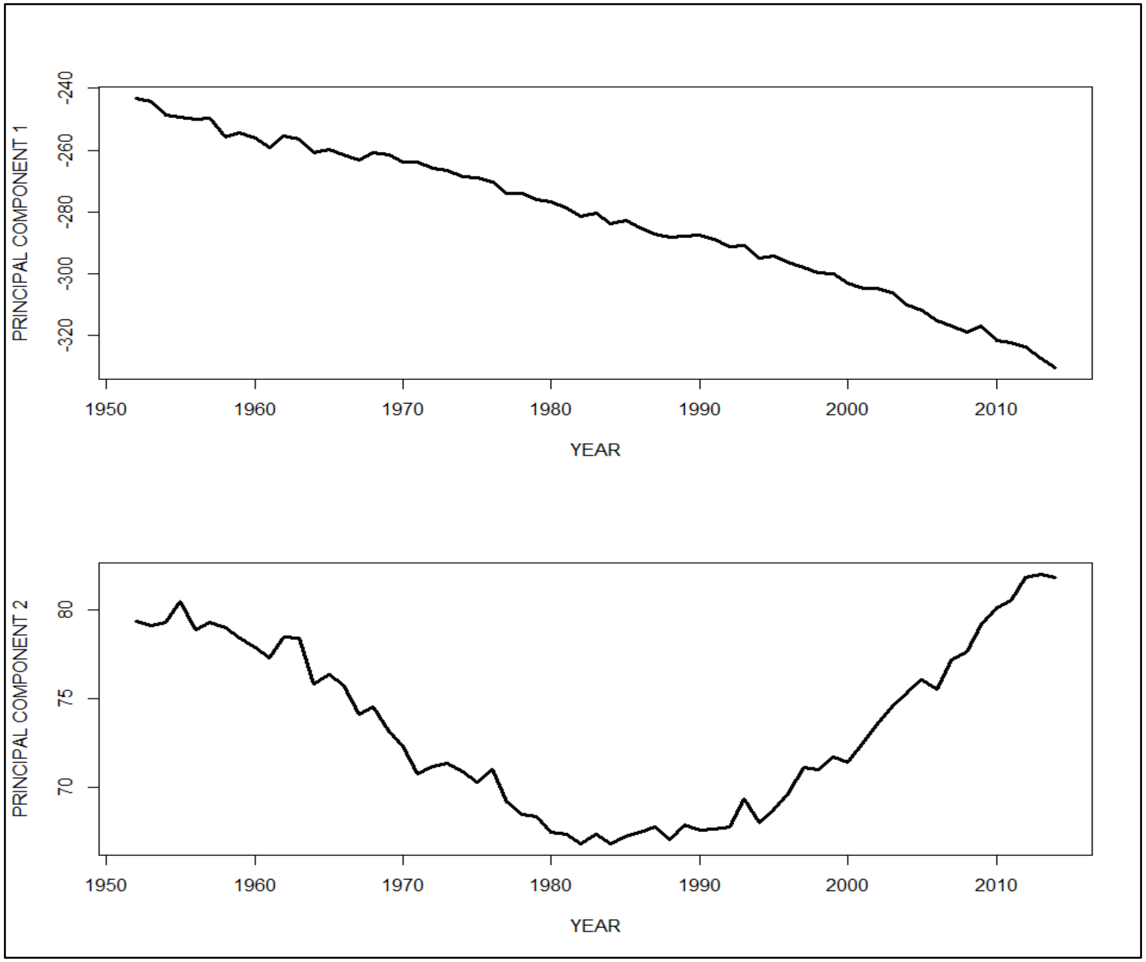
3. PRELIMINARY RESULTS

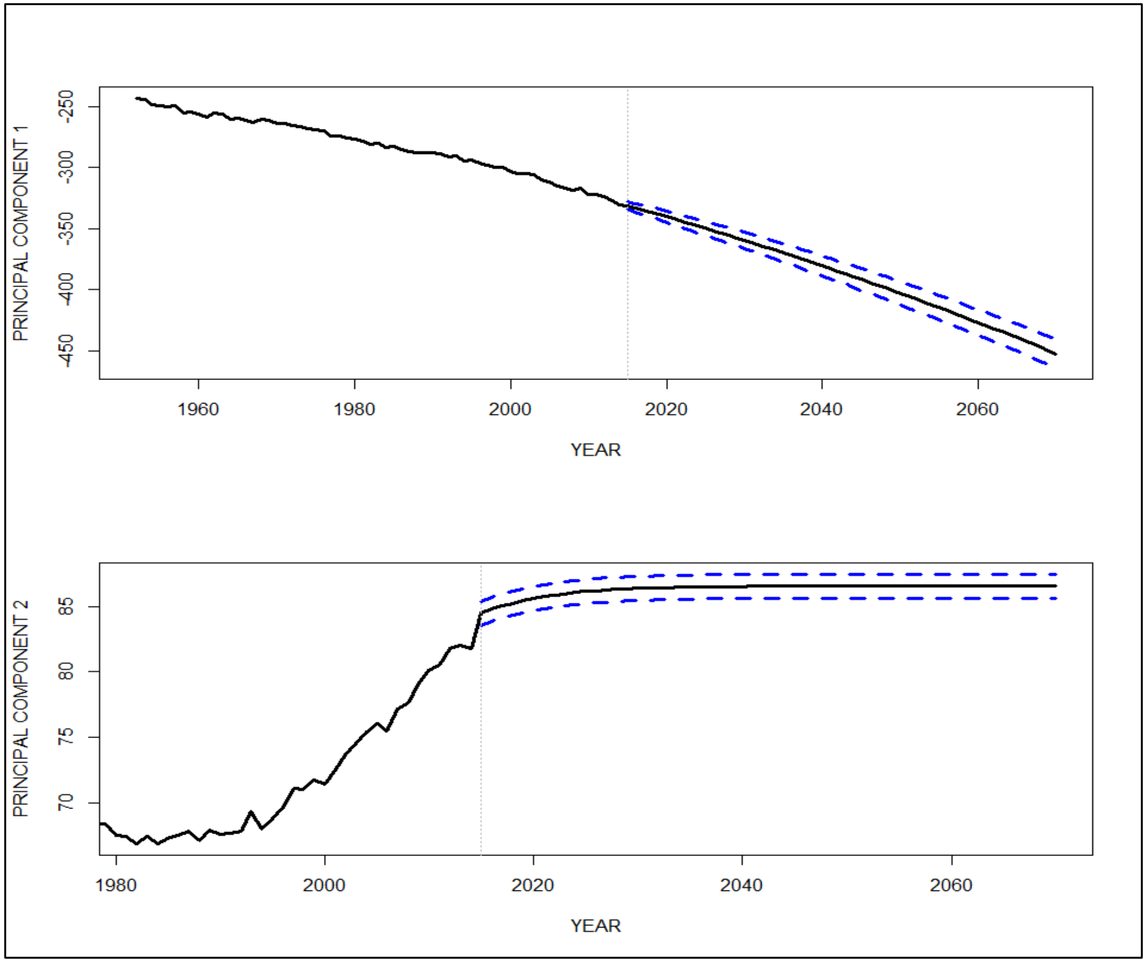
Figure 3: Historical course of the PC



Source: Own calculation and design

3. PRELIMINARY RESULTS

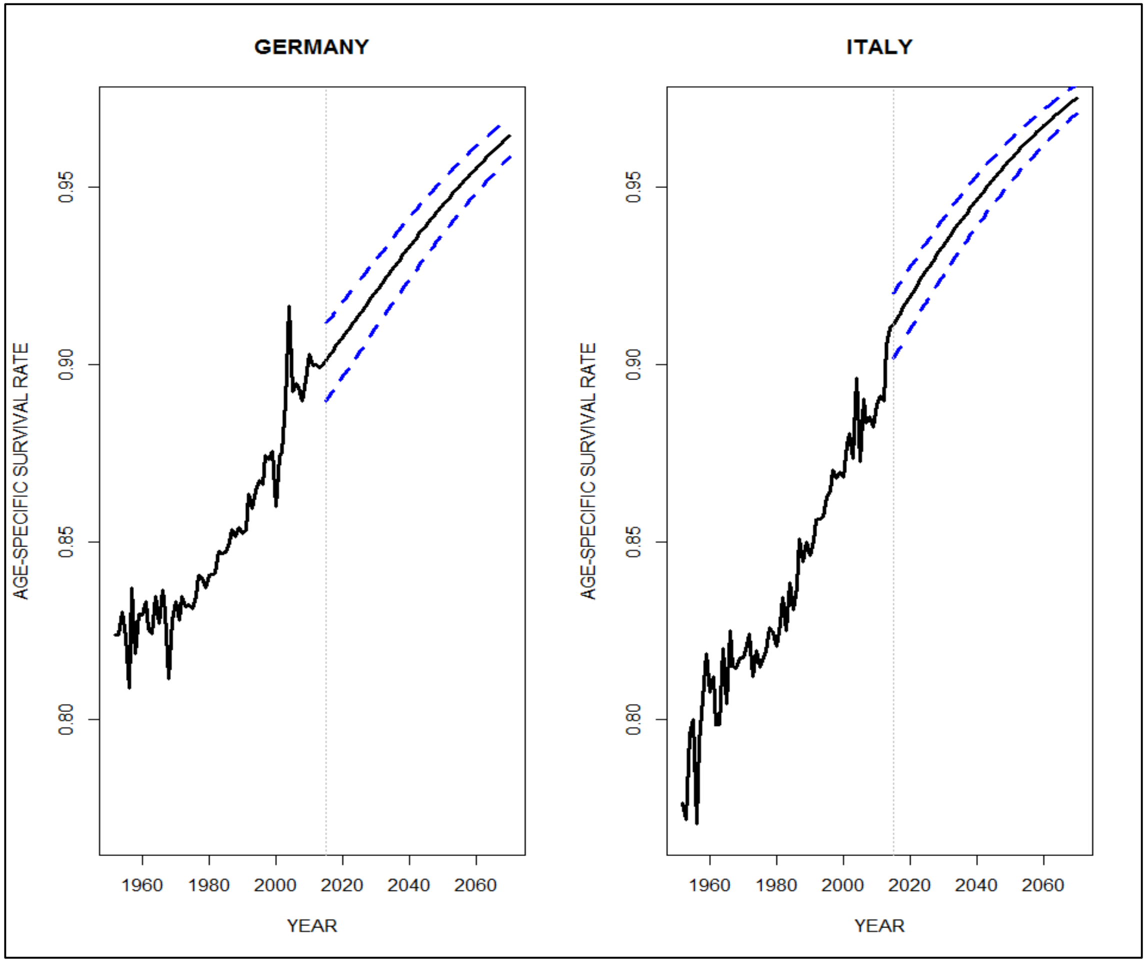
Figure 4: Forecast of PC 1 and PC 2 until 2070



Source: Own calculation and design

3. PRELIMINARY RESULTS

Figure 5: Forecast for 85-year-old men in Germany and Italy



Source: Own calculation and design

Conclusion:

- modeling approach for long-term forecasting of age-specific survival probabilities
- quantifying predictive intervals for the ASSR as well as the median life spans for males and females
- Quasi-reduction to a two-variable model
- easy structure and manageable need for data
- regular updating of mortality forecasts and life tables
- Takes care of overall stochasticity in mortality appropriately

Limitations:

- future mortality in the very high ages cannot be estimated conclusively
- no clear answer to the question, whether there is an upper limit in life span
- model cannot predict unexpected and not observed events like wars

Outlook:

- Derivation of PI for Median Life Spam from ASSR forecasts:

$$P(a) = \prod_{i=0}^{a} \pi_i$$

- bigger sample size of older population in the future will make data more reliable

Thanks for your attention!
Questions or remarks?

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