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Self-Insurance, Self-Protection, and Increased Risk Aversion: An Intertemporal Reinvestigation

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Agenda

1. Motivation
2. Literature Review
3. The Basic Model
4. The Impact of Endogenous Savings
5. Conclusion

1. Motivation

- Many prevention decisions take the form of investing now to reduce a future risk exposure
  - Sprinkler system
  - Burglar alarm
- A suitable model for such preventative measures has to resemble the fact that current consumption is sacrificed in order to improve on the distribution of wealth in an upcoming period

- Does a more risk-averse individual always choose a higher level of risk-reducing effort?

2. Literature Review

- Ehrlich and Becker (1972) use the distinction between self-insurance and self-protection

- **Self-insurance/Loss reduction:**

  Effort undertaken to reduce the size of a loss

  Example: Sprinkler system

- **Self-protection/Loss prevention:**

  Effort undertaken to reduce the probability of a loss

  Example: Burglar alarm

- Dionne & Eeckhoudt (1985):

- Monotonic relationship between risk aversion and self-insurance
- Ambiguous relationship between risk aversion and self-protection

Self-insurance and self-protection are rather distinct measures of risk reduction

Robustness of monotonic relationship between risk aversion and self-insurance:

Briys and Schlesinger (1990)
-> State-dependent utility;
-> Background risk

Explanations for the ambiguous relationship between risk aversion and self-protection:

Jullien et al. (1999) -> endogenous threshold p

Eeckhoudt and Gollier (2005) -> Prudence

Chiu (2000)

3. The Basic Model

- Self-insurance

$$\max_{y}\{u(w_1 - c(y)) + \delta\big(pu(w_2 - l(y)) + (1-p)u(w_2)\big)\}$$

- w_1 wealth today
- w_2 wealth tomorrow
- Y level of self-insurance activity
- c(y) cost of self-insurance, c‘>0, c‘‘>0
- P probability of loss
- l(y) size of the loss, l‘<0, l‘‘>0

Technical Requirement: Pratt (1964)

THEOREM 1: *Let $r_i(x)$, $\pi_i(x,\tilde{z})$, and $p_i(x)$ be the local risk aversion, risk premium, and probability premium corresponding to the utility function u_i, $i=1,2$. Then the following conditions are equivalent, in either the strong form (indicated in brackets), or the weak form (with the bracketed material omitted).*

(a) $r_1(x) \geqq r_2(x)$ *for all* x [*and* $>$ *for at least one* x *in every interval*].

(b) $\pi_1(x,\tilde{z}) \geqq [>] \pi_2(x,\tilde{z})$ *for all* x *and* $\tilde{z}$.

(c) $p_1(x,h) \geqq [>] p_2(x,h)$ *for all* x *and all* $h>0$.

(d) $u_1(u_2^{-1}(t))$ *is a* [*strictly*] *concave function of* t.

(e) $\dfrac{u_1(y)-u_1(x)}{u_1(w)-u_1(v)} \leqq [<] \dfrac{u_2(y)-u_2(x)}{u_2(w)-u_2(v)}$ *for all* v, w, x, y *with* $v<w \leqq x<y$.

The same equivalences hold if attention is restricted throughout to an interval, that is, if the requirement is added that $x, x+\tilde{z}, x+h, x-h, u_2^{-1}(t), v, w,$ *and* y, *all lie in a specified interval.*

- FOC

$$c'(y)u'(w_1 - c(y)) = -\delta p l'(y) u'(w_2 - l(y))$$

$$\Rightarrow y_u$$

v'(w) = k'(u(w)) u'(w)

- v=k(u), k'>0, k''<0

- FOC for agent v

$$EV'(y) = -c'(y)k'(u(w_1 - c(y)))u'(w_1 - c(y))$$
$$- \delta p l'(y) k'(u(w_2 - l(y)))u'(w_2 - l(y))$$

- Now, by substituting in the FOC for agent u

$$
\begin{aligned}
EV'(y_u) &= -c'(y_u)k'(u(w_1 - c(y_u)))u'(w_1 - c(y_u)) \\
&\quad - \delta p l'(y_u)k'(u(w_2 - l(y_u)))u'(w_2 - l(y_u)) = \\
&= c'(y_u)u'(w_1 - c(y_u))\underbrace{\left[k'(u(w_2 - l(y_u))) - k'(u(w_1 - c(y_u)))\right]}_{\text{Positive if and only if } \mathbf{w_1\text{-}c(y_u) > w_2\text{-}l(y_u)}}
\end{aligned}
$$

- The first derivative at y_u is positive as long as u's period-1 consumption exceeds u's period-2 consumption in the loss state.

The Basic Model

- Therefore, the following proposition holds:

Proposition 1:
Agent v invests more in self-insurance than agent u if and only if u's period-1 consumption exceeds u's period-2 consumption in the loss state.

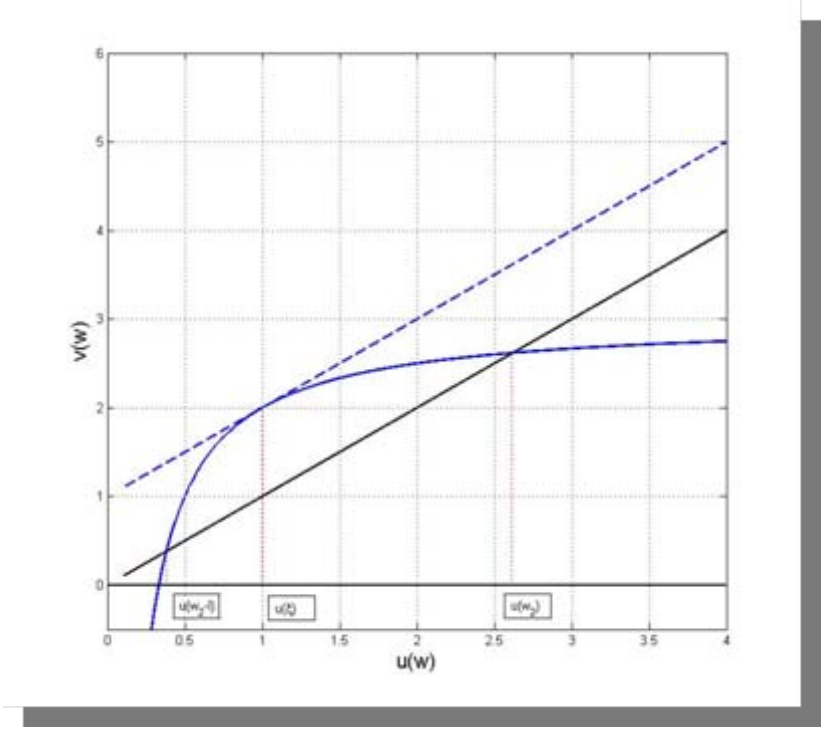
- Intuition: Consumption smoothing versus risk management incentives

- Self-protection

$$\max_{x}\left\{u(w_1 - c(x)) + \delta\left(p(x)u(w_2 - l) + (1 - p(x))u(w_2)\right)\right\}$$

- w_1 wealth today
- w_2 wealth tomorrow
- X level of self-protection activity
- c(x) cost of self-protection, c‘>0, c‘‘>0
- p(x) probability of loss, p‘<0, p‘‘>0
- L size of the loss

- Let $\xi \in (w_2\text{-}l, w_2)$ be such that $k'(u(\xi)) = 1$.



- The following proposition holds:

Proposition 2:
Agent v invests more in self-protection than agent u if and only if u's period-1 consumption exceeds ξ.

- Intuition: Consumption smoothing versus risk management
- Contrary to Dionne and Eeckhoudt (1985), self-insurance and self-protection react qualitatively very similarly to an increase in risk aversion in our model.

Endogenous Savings

- In a two-period model it is natural to discuss the impact of saving:

> If individuals utilize savings to smooth consumption over the life-cycle, will the effect of increased risk aversion be different or more similar to the static case?

- Self-insurance

$$\max_{y,s}\left\{u(w_1 - c(y) - s) + \delta\left(pu(w_2 - l(y) + (1+r)s) + (1-p)u(w_2 + (1+r)s)\right)\right\}$$

- s savings
- r interest rate

Endogenous Savings

- FOCs

$$c'(y)u'(z_1) = -\delta p l'(y) u'(z_{2L})$$

$$u'(z_1) = \delta(1+r)\left(p u'(z_{2L}) + (1-p) u'(z_{2N})\right)$$

- z_1 period-1 consumption
- z_{2L} period-2 consumption in loss state
- z_{2N} period-2 consumption in no-loss state

- It can be shown that saving and self-insurance are substitutes, i.e.,

$$\frac{dy}{ds} = -\frac{EU_{ys}}{EU_{yy}} = -\frac{c'u''(z_1) - \delta(1+r)\, p l' u''(z_{2L})}{EU_{yy}} < 0$$

Endogenous Savings

- Let α be a parameter associated with the degree of risk aversion, implicit differentiation of focs gives

$$\frac{dy}{d\alpha} = \frac{1}{D}\Big(\underbrace{-EU_{ss}EU_{y\alpha}}_{\text{Direct effect}} + \underbrace{EU_{ys}EU_{s\alpha}}_{\text{substitution effect}}\Big)$$

- Looks like effect should be ambiguous…

The Basic Model

- The following proposition holds:

Proposition 3:
Under endogenous saving, stronger risk aversion leads to more self-insurance.

- Intuition: Consumption smoothing is taken care of by endogenous savings

Endogenous Savings

- Self-protection

$$\max_{x,s}\left\{u(w_1 - c(x) - s) + \delta\Big(p(x)u(w_2 - l + (1+r)s) + (1-p(x))u(w_2 + (1+r)s)\Big)\right\}$$

Endogenous Savings

- We find that

Proposition 4:
Under endogenous saving, stronger risk aversion leads to more self-protection if and only if the loss probability is below an endogenously determined threshold.

- Intuition: By endogenously determining the optimal level of saving, individuals take care of consumption smoothing motives and self-protection is entirely devoted to risk mitigation.

Endogenous Savings

Result:

- As soon as savings are endogenized, self-insurance and self-protection behave as if modeled in a static sense!
- The results developed by Dionne and Eeckhoudt (1985) are robust to the consideration of an intertemporal model if and only if savings are endogenous!

Conclusion

- Realistic models of intertemporal prevention should use an investment-based view!
- In a two-period model, if individuals use self-insurance or self-protection alone to mitigate risks, these two devices behave very similarly:

  Stronger risk aversion is associated with more risk reducing behavior, if and only if current consumption is above an endogenous threshold, i.e., if and only if more risk reduction is "affordable".
- As soon as savings are endogenized, stronger risk aversion is associated with more self-insurance; and stronger risk aversion leads to more self-protection, if and only if the loss probability is below an endogenous threshold.
- Whether individuals take a separate risk reduction decision or a joint risk reduction and saving decision makes a large difference!

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