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Scenario-based Capital Requirements for the Interest Rate Risk of Insurance Companies

Agenda

Motivation

Model & Results

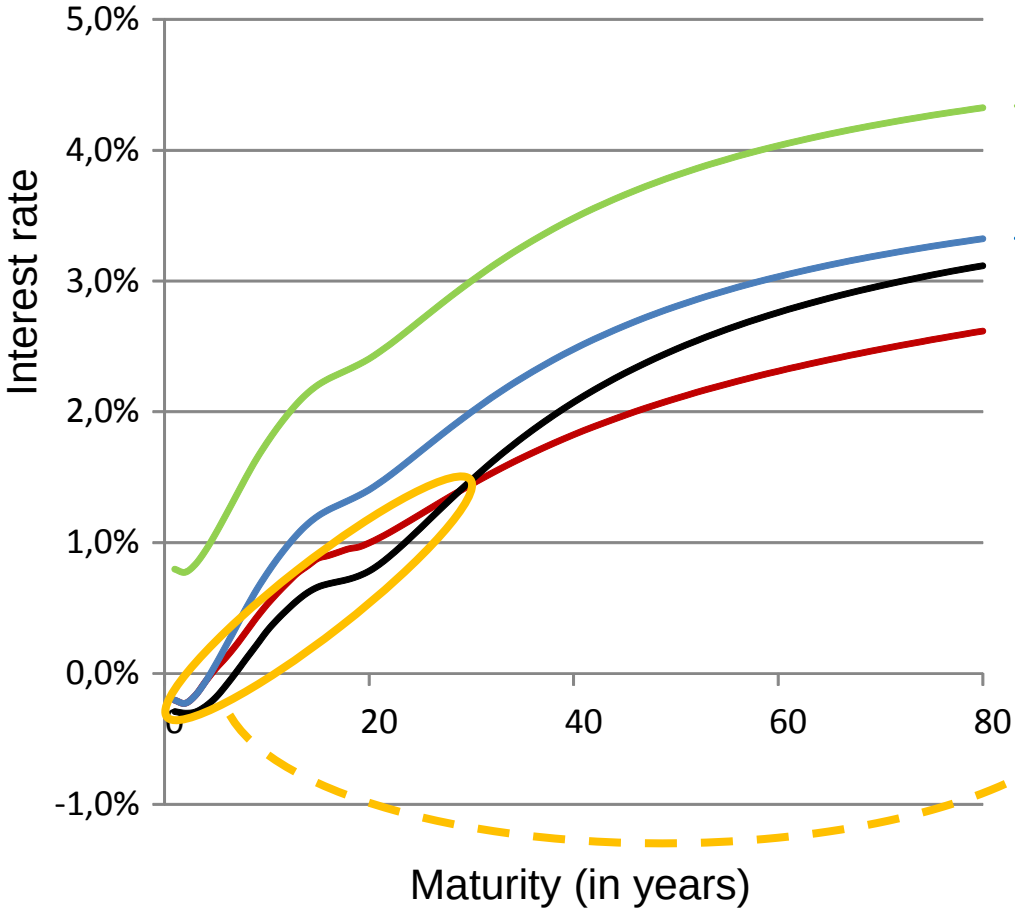
Conclusion

Interest Rate Risk of Insurance Companies

- … is a serious threat in particular for life insurers
- Duration gap > **10 years** on average for life insurers in Austria, Germany, Lithania, Sweden (cf. EIOPA report on insurance stress test 2014)
- Consequently, interest rate risk has a considerable impact on their required capital
- Regulatory capital requirements should accurately measure interest rate risk in order to set the right risk management incentives

Solvency II standard formula for interest rate risk

Stress scenarios
- Defined in terms of relative changes
- Shall reflect 1-in-200 year event



Upward scenario (at least +1%)

Base case (per Nov 2015)

Base case (per June 2016)

Downward scenario (Negative yields are not decreased further)

In 2016, EIOPA's base case yield curves have outrun the **previous month**'s downward stress scenario (for some maturities) in 9 of 12 monthly reports

Objective:
Better model for measuring interest rate risk

Model shall fulfill the following properties:

- Estimate the 99.5%-Value-at-Risk over a one-year time horizon
- Pass backtesting against historical yield curve movements
- Reflect insurers' exposure to curvature changes of the yield curve
- Account for a lower bound to interest rates (economic costs of storing cash)
- Pragmatic approach, e.g. based on a few scenarios

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Measuring interest rate risk – possible starting points

i. Affine factor models (e.g. Cox Ingersoll Ross, 1985)
   - Focus on shifts of the yield curve (short rate)
   - Bad performance in forecasting the yield curve (cf. Duffee, 2002)

not used

ii. Model of Svensson (1994):
   - Spot rate at time t for maturity s represented as:

$$r_t(s) = \beta_{1,t} + \beta_{2,t}\frac{1-e^{-\lambda_t s}}{\lambda_t s} + \beta_{3,t}\left(\frac{1-e^{-\lambda_{1,t} s}}{\lambda_{1,t} s} - e^{-\lambda_{1,t} s}\right) + \beta_{4,t}\left(\frac{1-e^{-\lambda_{2,t} s}}{\lambda_{2,t} s} - e^{-\lambda_{2,t} s}\right)$$

$\theta_t = (\beta_{1,t}, \beta_{2,t}, \beta_{3,t}, \beta_{4,t}, \lambda_{1,t}, \lambda_{2,t})$ is vector of time-varying parameters (Diebold and Li, 2006)

   - Has been used to model the Value-at-Risk over short time horizons (Caldeira et al. 2015)
   - Incorporation of a lower bound for interest rates possible, but numerically challenging (cf. Eder et al., 2014)

Measuring interest rate risk – possible starting points

iii. Two-step approach („Exponential“)

<u>Step 1:</u> Model the interest rate for some maturities (e.g. s = 1, 5, 10, 20, 30 years):

$$r_t(s) = e^{\theta_t(s)} + r^{\min}(\tau)$$

Lower bound to interest rate with maturity s (going forward: -3%)

$\theta_t(s)$ are time-varying parameters

<u>Step 2:</u> Use the Svensson model to extrapolate the 5 interest rates to the complete term structure

Development of parameter vector θ_t over time

Challenge: Future movement of yield curve may depend on its current level and curvature

Solution: Model θ_t by an autoregressive (AR) or vector autoregressive (VAR) process:

Lag order

$$\Delta\theta_t = \mu + \sum_{k=1}^{p} \Gamma_k \cdot \theta_{t-k} + \eta_t$$

Vector of constants

Transition matrix (AR process: diagonal matrix)

Vector of residuals
Challenge: may exhibit autocorrelation, time-varying volatility and correlations

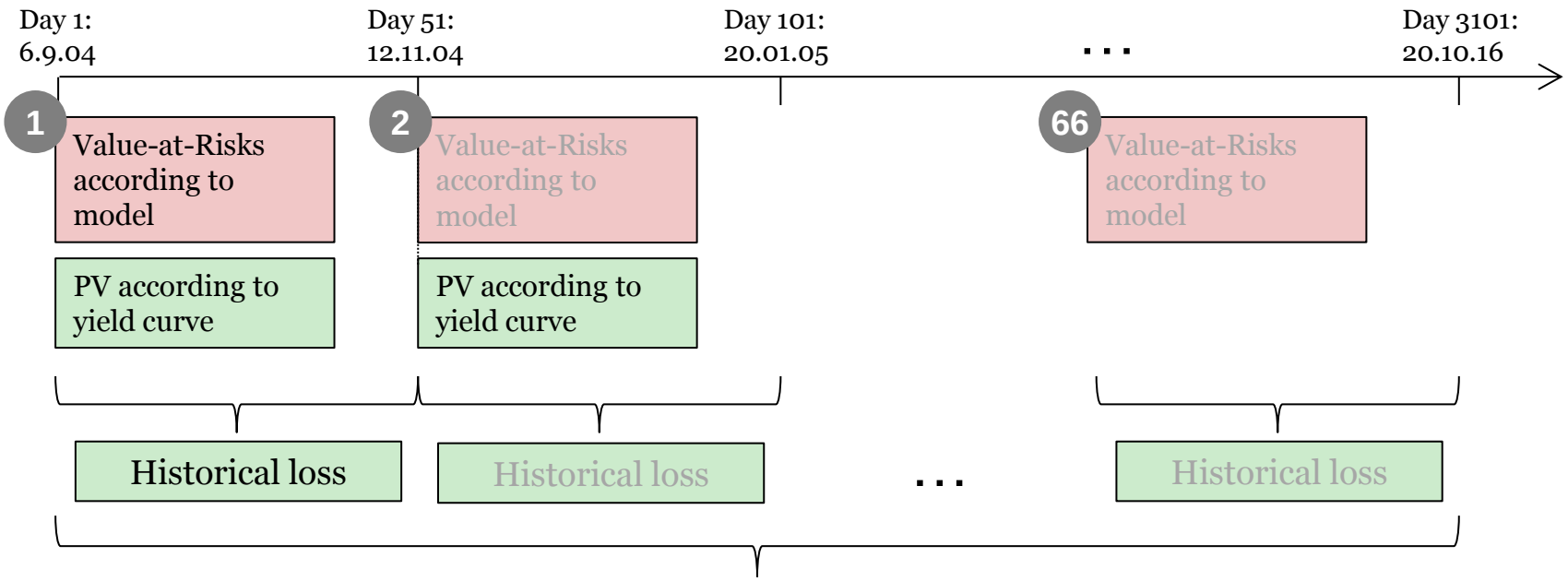
Solution: Model η_t by the dynamic conditional correlation (DCC) model (Engle, 2002)

→ 4 models: Svensson / Exponential, combined with VAR / AR process

Calibration

- Use daily yield curve data provided by ECB from 6.9.2004 – 31.10.2016 (3110 days)
- Remove drift from $\Delta\theta_t$ –process (e.g. the observed reduction in interest rates may not continue in future)
- Determine lag order of (vector) autoregressive process according to Hannan-Quinn criterion
- Estimate VAR and AR models based on OLS regression per equation
- Choose marginal distributions for η_t according to Akaike information criterion (either normal or generalized hyperbolic)
- Estimate DCC model (R rmgarch package by Ghalanos 2015)

Backtesting against historical movements - Example: VaR with 50 days holding period



- Repeat this for all disjunct 50-day time windows (➔ 66 VaR's and historical losses)
- Hit rate: Count how often „Historical loss > Value-at-Risk", divided by 66

- Carry out backtesting for 100 asset-liability portfolios (each with 5 cash flows whose amounts and maturities are randomly chosen)

Backtesting - Results

- Hit rates for
  - holding periods 1, 15, 30 and 50 days
  - confidence levels 0.5%, 5%, 10%
  - all 4 models

Accross the 100 portfolios

Model	Holding period (in days)	Confidence level 0.50% Average	0.50% Std Dev.	5% Average	5% Std Dev.	10% Average	10% Std Dev.
Svensson-AR	1	0.20%	0.22%	1.90%	1.53%	4.23%	3.04%
	15	0.41%	0.51%	4.29%	2.27%	8.68%	4.05%
	30	0.26%	0.66%	5.82%	3.48%	10.57%	5.41%
	50	0.65%	1.41%	5.24%	4.17%	12.74%	6.36%
Svensson-VAR	1	0.10%	0.09%	0.83%	0.41%	1.98%	0.64%
	15	0.12%	0.27%	1.10%	0.87%	2.73%	1.52%
	30	0.03%	0.22%	0.85%	1.14%	2.74%	2.66%
	50	0.02%	0.16%	0.61%	1.20%	3.65%	2.29%
Exp-AR	1	0.56%	0.32%	4.05%	0.49%	8.18%	0.70%
	15	0.73%	0.53%	4.23%	1.06%	8.38%	1.74%
	30	0.50%	0.76%	4.41%	2.22%	8.26%	2.59%
	50	0.19%	0.57%	4.32%	1.93%	8.42%	3.43%
Exp-VAR	1	0.55%	0.33%	4.03%	0.50%	8.23%	0.61%
	15	0.77%	0.61%	4.07%	1.49%	8.66%	1.95%
	30	0.85%	1.06%	4.23%	2.44%	8.23%	2.90%
	50	0.27%	0.65%	5.48%	2.49%	9.87%	3.89%

For short time horizons, hit rate ≈ confidence level

Best model

By increasing time horizon, hit rate does not systematically increase (stable risk aggregation over time)

Simplification to scenarios

- Result so far: We can generate simulations of θ_t which can be translated into simulations of the yield curve, or of the discount factors
- Idea for simplification:

(1) Portfolio losses are linear in discount factors:

Discount factors $X_1 = \left(e^{-r_{1\text{ year}}(\tau_1)}, \ldots, e^{-r_{1\text{ year}}(\tau_K)}\right)^T,$

Insurer's future surpluses: $S = (S_{\tau_1}, \ldots, S_{\tau_K})$

$$\mathrm{VaR}_{1-\alpha}(1\text{ year}) = -\left(q_\alpha(X_1^T \cdot S) - X_0^T \cdot S\right)$$

(2) Transform simulated discount factors via PCA

→ Elicit stressed yield curves

Recipe for scenario-based Value-at-Risk

- Regulator determines stressed yield curves according to absolute changes shown below
- Insurer recalculate the own funds in each scenario

		Maturity				
		1 year	5 years	10 years	20 years	30 years
PC1	A	0.4%	1.0%	1.7%	3.5%	6.4%
	B	-0.4%	-0.9%	-1.3%	-1.9%	-1.9%
PC2	A	-0.2%	-0.4%	-0.6%	-0.1%	0.1%
	B	0.3%	0.7%	1.0%	0.2%	-0.2%
PC3	A	-0.2%	-0.3%	-0.2%	0.3%	-0.1%
	B	0.0%	0.1%	0.1%	-0.1%	0.0%
PC4	A	-0.3%	-0.3%	0.1%	0.0%	0.0%
	B	0.4%	0.4%	-0.1%	0.0%	0.0%
PC5	A	1.1%	0.0%	0.0%	0.0%	0.0%
	B	-0.8%	0.0%	0.0%	0.0%	0.0%

$$VaR_1 = \max\{Loss\ in\ scen.\ A; Loss\ in\ scen.\ B\}$$

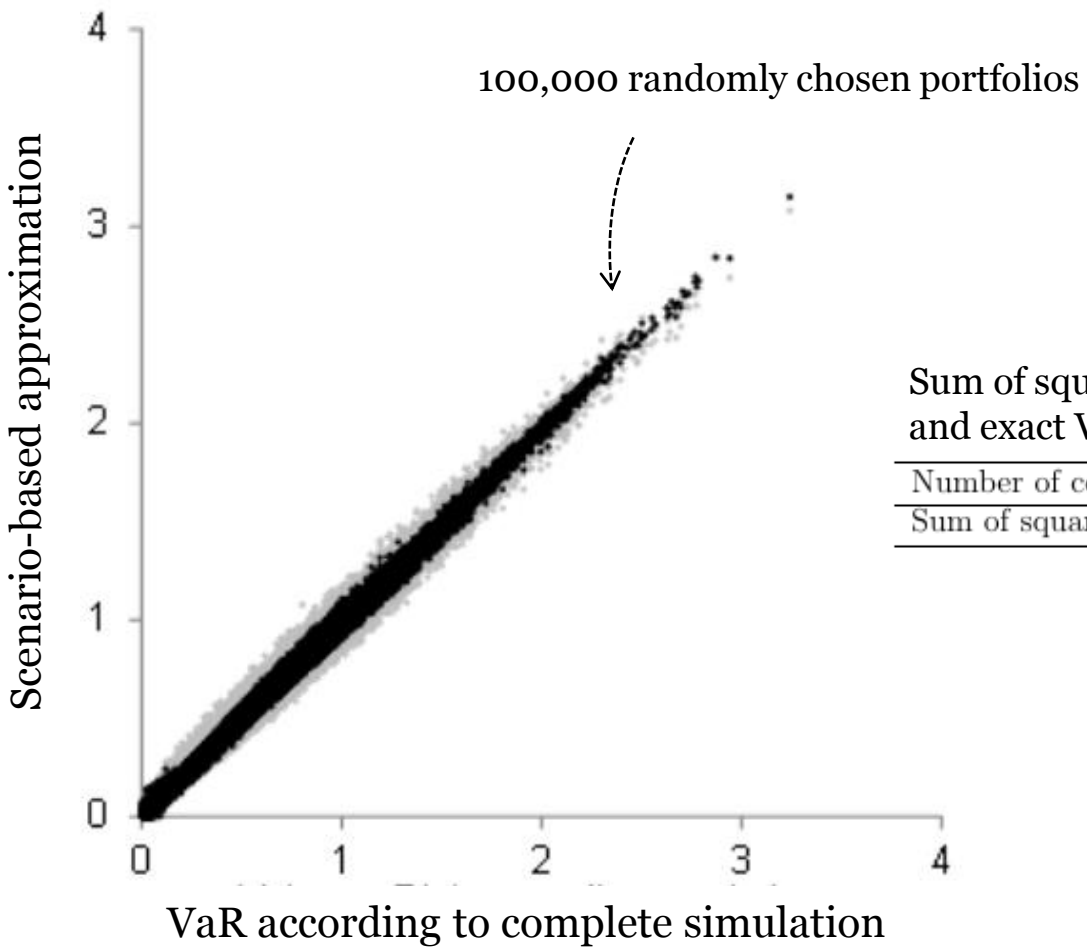
$$VaR_2 = \max\{Loss\ in\ scen.\ A; Loss\ in\ scen.\ B\}$$

...

- Aggregate the Value-at-Risks by a square-root formula :

$$\mathrm{VaR}_{1-\alpha}(1\text{ year}) \approx \sqrt{\sum_{k=1}^{\bar{K}} \mathrm{VaR}_k^2}$$

Backtest of scenario-based method



Sum of squared differences between scenario-based and exact Value-at-Risk

Number of components, K	1	2	3	4	5
Sum of squares	123.2	53.8	53.5	53.1	53.1

Using more than the first two components hardly improves goodness-of-fit
(i.e., 4 scenarios are needed)

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- *Good news:* 4 scenarios suffice to determine the Value-at-Risk for interest rate risks almost as exact as a an internal model

- *Bad news:* Solvency II standard formula for interest rate risks appears too optimistic also in regard of upward movements
  (at least in the current low yield environment, where EIOPA's relative stress factors have a small absolute impact)