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Is Fair Pricing Possible? An Analysis of Participating Life Insurance Portfolios

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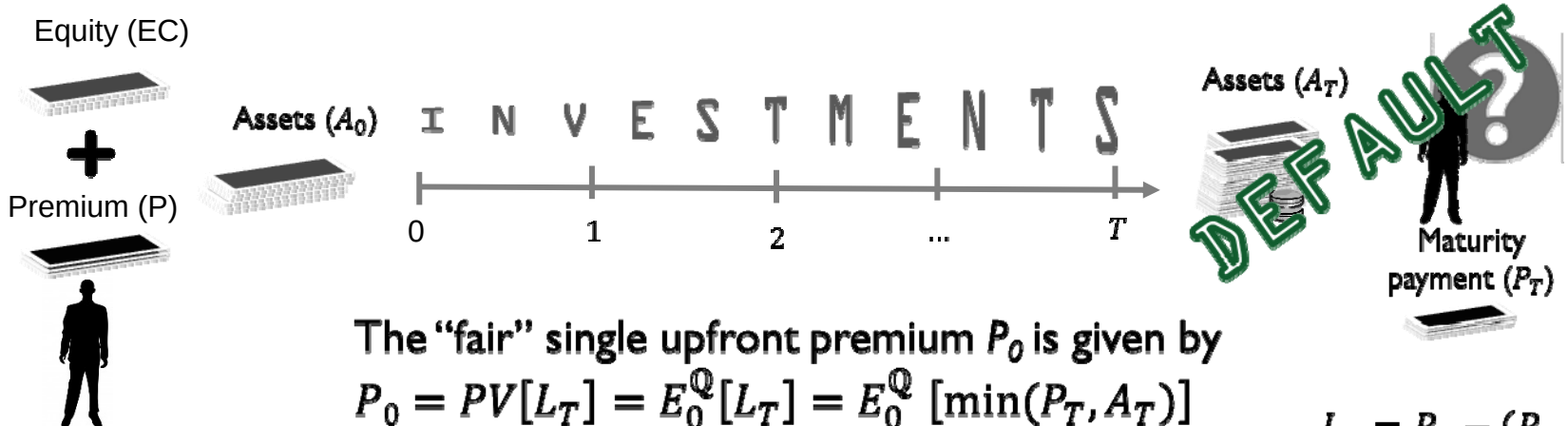
Structure

1. **Motivation**
2. **Model Description**
3. **Numerical Illustration**
4. **Conclusion**

1. Motivation

Up today the pricing models that include insurer's default risk in the participating life insurance literature…

Cf. Briys & de Varenne (1997), Grosen & Jorgensen (2002), Ballotta et al. (2006), Ballotta (2009), Bacinello (2001), Tanskanen & Lukkarinen (2003), Schmeiser & Wagner (2014), and many more



The "fair" single upfront premium P_0 is given by

$$P_0 = PV[L_T] = E_0^{\mathbb{Q}}[L_T] = E_0^{\mathbb{Q}}\,[\min(P_T, A_T)]$$

$$\text{NPV} = PV[L_T] - P_0 = 0$$

$$L_T = P_T - (P_T - A_T)^+$$

$$L_T = P_T - D_T$$

D_T: Default cost

1. Motivation - Contribution

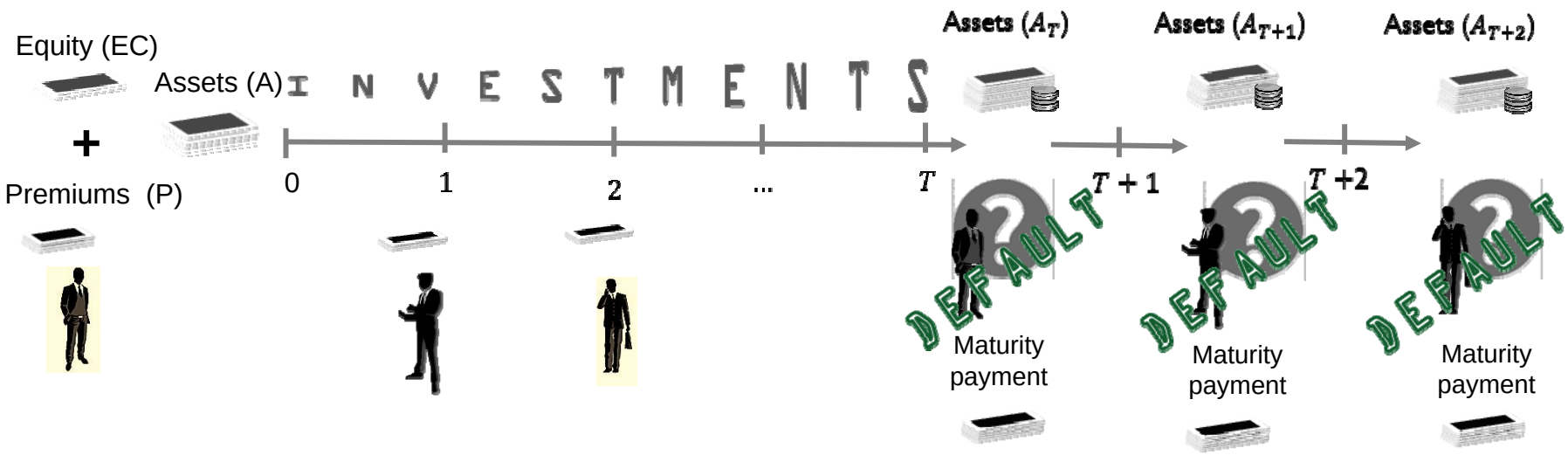
In insurance practice we typically have

… different contracts' starting points / maturity dates

… the contracts are typically pooled in one legal entity

… sharing of default risk

There is very little research so far that tries to capture these points (cf. Hansen & Miltersen 2002, Gerstner et al. 2008, Ibragimov et al. 2010, Doskeland & Nordahl 2008, Gollier 2008)



1. Motivation

Research question 1:

Is it possible to simultaneously charge fair premiums (*NPV* = 0) to all policyholders?

NPV in formal terms:

$$NPV_P = \sum_{i=1}^{n} NPV_{i,0}$$

and

$$NPV_{i,0} = e^{-r_f \cdot \tau_i} \left(\Pi^P_{i,\tau_i} - P_i \right) = \Pi^P_{i,0} - e^{-r_f \cdot \tau_i} P_i$$

(Present value of the policyholder's expected payoff (policyholder generation i) minus present value of the premium paid by policyholder i)

1. Motivation

If only fair terms for the portfolio can be provided,

…. who pays more? Who pays less?

	T	T+1	T+2	Portfolio
PV premiums				
PV payoff				
Net present value	> 0	>= 0	<= 0	= 0

Portfolio is in total fairly priced

1. Motivation

Research question 2:

Are all policyholders encountering the same default risk?

	T	T+1	T+2
Premiums			
Default		DEFAULT	DEFAULT
Risk level	Low	Medium	High

Is it possible to ensure the same value for the default put option (ratio) to all policyholders / generation of policyholders?

1. Motivation

Summary:

FAIR PREMIUM FOR ALL GENERATIONS + **SAME DEFAULT RISK LEVEL FOR ALL GENERATIONS**

Remark

Structure

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2. Model description

Basic framework

Participating life insurance contract with cliquet-style interest rate guarantee

Mortality risk is not taken into account

Single premiums

Fixed time to maturity for all contracts

Two asset classes: Risky assets and riskless rate of return

Risky assets are modelled through a GBM

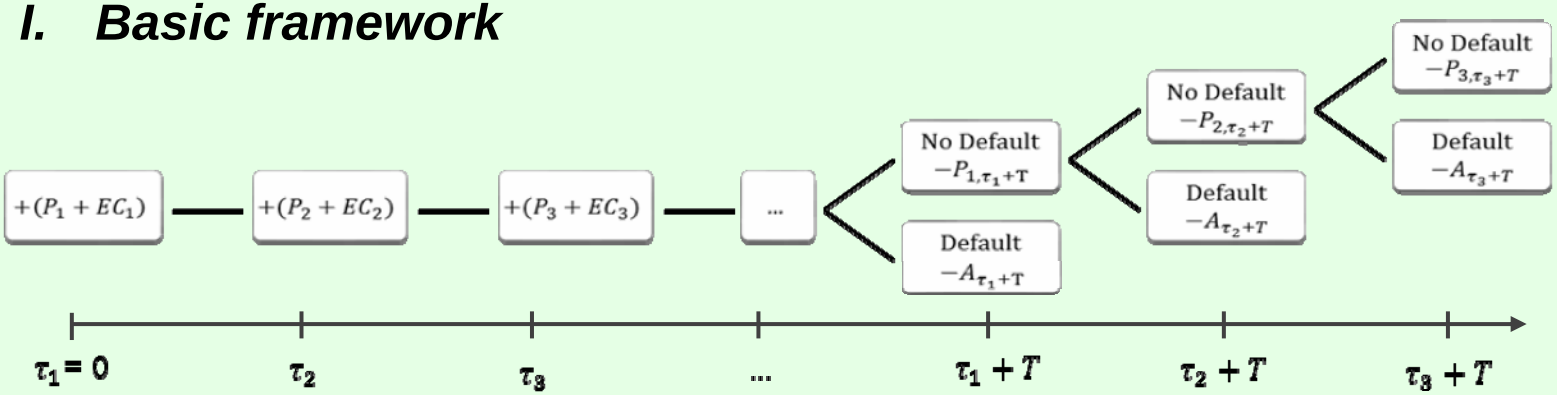
Risk management measures that can be taken by the insurer:

- **Equity capital contributions**
- **Asset Allocation**

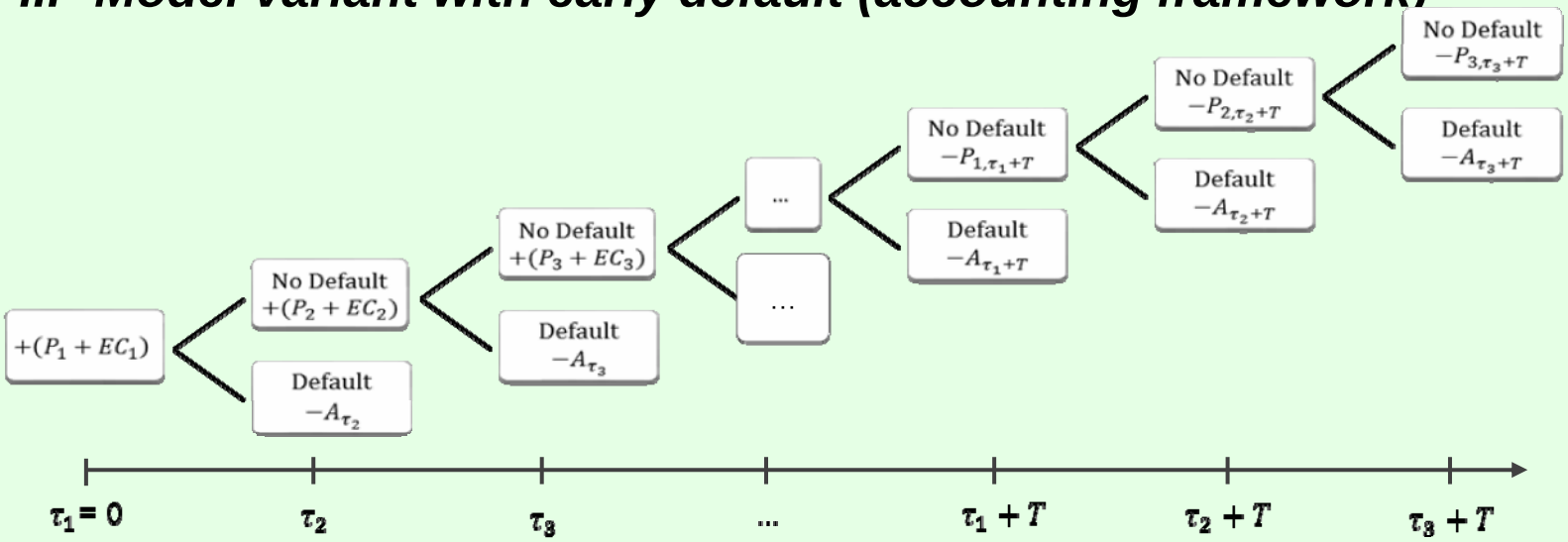
2. Model description

Pension scheme

I. Basic framework



II. Model variant with early default (accounting framework)



Life insurer

2. Model description

Formal structure:

Policyholder account of the i-th policyholder generation

$$P_{i,t} = \begin{cases} P_{i,t-1} \cdot (1 + r_{i,t}) & \text{if } \tau_i + 1 \leq t < \tau_i + T \\ 0 & \text{elsewhere} \end{cases}$$

$$r_{i,t} = max\,(g, \alpha \cdot r_{P,t})$$

$$r_{P,t} = (1 - \gamma) \cdot (e^{r_f} - 1) + \gamma \cdot \left(e^{\left(\mu - \frac{\sigma^2}{2}\right) + \sigma[W_t - W_{t-1}]} - 1 \right)$$

Evolution of the assets

$$A_t = \begin{cases} A_{t-1} \cdot (1 + r_P) + P_i + EC_i & \text{if } t = \tau_i < \tau_1 + T \\ A_{t-1} \cdot (1 + r_P) - L_{i,t} & \text{if } t = \tau_1 + T, ..., \tau_n + T \\ A_{t-1} \cdot (1 + r_P) & \text{otherwise} \end{cases}$$

2. Model description

Basic framework (I):

Insurer pursued its activities as long as

$$A_t > \widehat{P}_t = \sum_{i \in \Omega_t} P_{i,t}$$

holds at maturity. Thereby

$$\widehat{P}_t = \sum_{i \in \Omega_t} P_{i,t}$$

denotes the accumulated liabilities of the insurer. In case of a default, the cost of insolvency correspond to the difference between the accumulated liabilities and the available assets

$$D_t = \left(\widehat{P}_t - A_t\right)^+$$

Proportional insolvency costs for the i-th policyholder generation

$$D_{i,t} = (P_{i,t} - \lambda_{i,t} \cdot A_t)^+ \quad \text{for} \quad i \in \Omega_t$$

$$\lambda_{i,t} = \frac{P_{i,t}}{\sum_{i \in \Omega_t} P_{i,t}} \quad \text{for} \quad i \in \Omega_t$$

3. Model description

Basic framework (II):

Payoff when taken default risk into account

$$L_{i,t} = \begin{cases} P_{i,t} - D_{i,t} & \text{if } \widehat{P}_t \geq A_t \text{ and } \tau_1 + T \leq t \leq \tau_i + T \\ P_{i,t} & \text{if } \widehat{P}_t < A_t \text{ and } t = \tau_i + T \\ 0 & \text{otherwise} \end{cases}$$

Present value of the contracts of the i-th policyholder generation

$$\Pi_{i,0}^{P} = \sum_{t=\tau_1+T}^{\tau_i+T} E_0^{\mathbb{Q}}\left[L_{i,t}\right] = \sum_{t=\tau_1+T}^{\tau_i+T} E^{\mathbb{Q}}\left[e^{-r_f \cdot t} L_{i,t}\right]$$

Fair pricing takes place if the net present value is zero (for both stakeholder groups)

3. Model description

Basic framework (III):

Default put option ratio is defined as

$$\mathrm{DPO}_i = \frac{\Pi_{i,0}^{DPO}}{\Pi_{i,0}^{*P}}$$

Value of the default put option for the contracts of the i-th policyholder generation

$$\Pi_{i,0}^{DPO} = \sum_{t=\tau_1+T}^{\tau_i+T} E_0^{\mathbb{Q}}\left[D_{i,t}\right] = \sum_{t=\tau_1+T}^{\tau_i+T} E^{\mathbb{Q}}\left[e^{-r_f \cdot t} D_{i,t}\right]$$

And denotes the value of the contract(s) without default

$$\Pi_{i,0}^{*P} = E_0^{\mathbb{Q}}\left[P_{i,\tau_i+T}\right]$$

Accounting model: Please cf. working paper

Structure

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3. Numerical Illustration

Table 1: Fixed model parameters

Parameter	Variable	Value
Contract		
Number of generations in portfolio	n	3
Generation up-front premium $\forall i, i \in \{1, ..., n\}$	P	1.0 (C.U.)
Issuance date, $i \in \{1, ..., n\}$	τ_i	$i - 1$
Contract duration $\forall i, i \in \{1, ..., n\}$	T	3 (years)
Guaranteed annual interest rate $\forall i, i \in \{1, ..., n\}$	g	1.75%
Annual surplus participation rate $\forall i, i \in \{1, ..., n\}$	α	90%
Capital market conditions		
Continuous risk-free rate of return	r_f	3%
Continuous mean return of the risky assets	μ	5%
Volatility of the risky assets	σ	20%

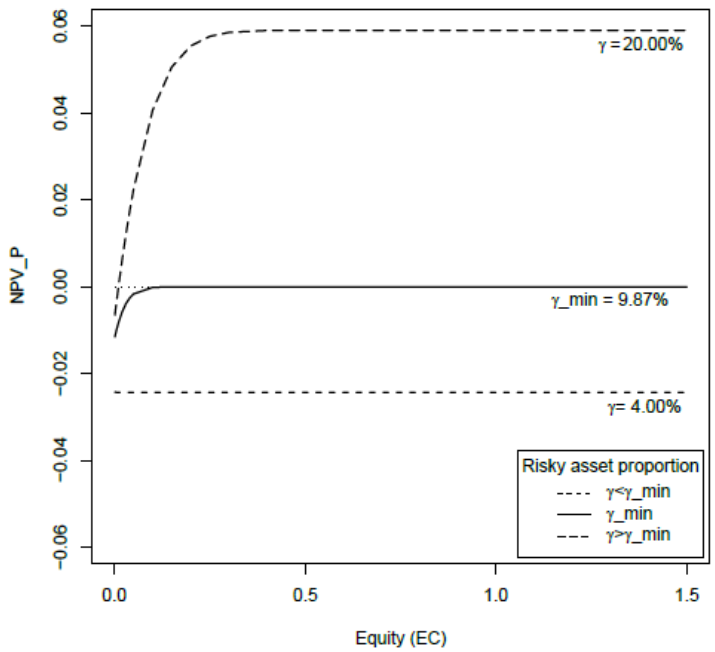
All customers in the portfolio have the same contract conditions.

3. Numerical Illustration

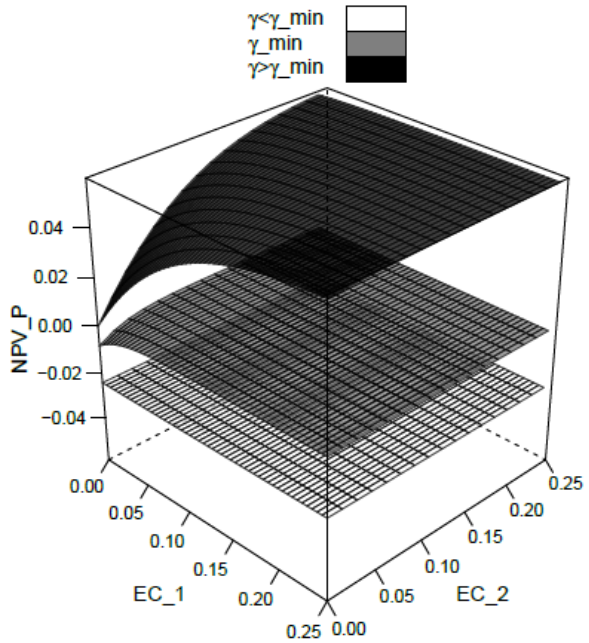
Basic framework (I):

It is not always possible to achieve a fair price for the portfolio as a whole

<u>i.e., combination of an investment strategy and equity contributions such $NPV_P=0$</u>



(a) Single-equity contributions



(b) Periodic equity contributions. Initial equity contribution $EC_0 = 0.01$

3. Numerical Illustration

Basic framework (II):

Fair pricing and equal default ratios are possible (for the portfolio as a whole and for each generation) only in the case without default

Table 3: Combinations of minimum investment shares in risky assets and minimum equity contribution such that the insurance portfolio is fairly priced in total.

	Single-equity contribution at t=0			Periodic equity contributions at issuances		
Variable	$P_i =$	$P_i \nearrow$	$P_i \searrow$	$P_i =$	$P_i \nearrow$	$P_i \searrow$
Risky Asset share						
$\gamma_{min}(\%)$	9.87	9.88	9.87	9.87	9.88	9.87
Equity contributions						
EC_0	0.1698	0.2618	0.1308	0.0300	0.0300	0.0300
EC_1	0.0000	0.0000	0.0000	0.3939	0.4193	0.3973
EC_2	0.0000	0.0000	0.0000	0.6916	0.5336	0.5034
Policyholders NPVs[a]						
NPV_{1,τ_1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
NPV_{2,τ_2}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
NPV_{3,τ_3}	0.0000	-0.0001	0.0000	0.0000	0.0000	0.0000
Shareholders NPV						
NPV_{EC}	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000

3. Numerical Illustration

Basic framework (III):

Fair pricing and equal default ratios are possible (for the portfolio as a whole and for each generation) only in the case without default

Table 3: Combinations of minimum investment shares in risky assets and minimum equity contribution such that the insurance portfolio is fairly priced in total.

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Variable	$P_i =$	$P_i \nearrow$	$P_i \searrow$	$P_i =$	$P_i \nearrow$	$P_i \searrow$
Risky Asset share						
$\gamma_{min}(\%)$	9.87	9.88	9.87	9.87	9.88	9.87
Shareholders NPV						
NPV_{EC}	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
Policyholders DPO[b]						
DPO_1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DPO_2	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000
DPO_3	0.0001	0.0002	0.0001	0.0000	0.0000	0.0000

$P_i =$: All generations pay the same premium 1 C.U. at issuance $P_i \nearrow$: Premiums paid by each new generation increases by 0.5 C.U. $P_i \searrow$: Premiums paid by each generation decreases by 0.25 C.U.

[a] NVP: Net present value. $NPV \simeq 0$ if $|NPV| < 10^{-4}$. $NPV_{EC} \simeq -NPV_P$.

[b] DPO: Default ratio. $DPO_i = 1$ corresponds to 1×10^{-2}. $DPO \simeq 0$ if $|DPO| < 10^{-6}$.

3. Numerical Illustration

Basic framework (IV):

The portfolio can be fairly priced, but NPVs for different policyholder generations and corresponding default ratios vary

Table 4: Strategies of single-equity contributions for different shares of investment in risky assets. Net present values and default ratios when all policyholders pay $P = 1$ C.U.

		Policyholders			Insurer	Default ratios		
$\gamma(\%)$	EC_0	NPV^a_{1,τ_1}	NPV^a_{2,τ_2}	NPV^a_{3,τ_3}	NPV^{ab}_{EC}	DPO^c_1	DPO^c_2	DPO^c_3
9.87	0.1698	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
10.00	0.0730	0.0001	-0.0000	-0.0001	0.0000	0.0144	0.0240	0.0273
15.00	0.0147	0.0026	-0.0003	-0.0025	0.0000	0.7248	0.8630	0.9071
20.00	0.0105	0.0057	-0.0004	-0.0057	0.0000	1.4241	1.6354	1.7017
25.00	0.0091	0.0090	-0.0004	-0.0091	0.0000	2.1219	2.3885	2.4708
30.00	0.0086	0.0124	-0.0006	-0.0125	0.0000	2.8165	3.1309	3.2239
50.00	0.0092	0.0259	-0.0009	-0.0265	0.0000	5.5389	5.9824	6.0800
80.00	0.0122	0.0465	-0.0017	-0.0474	0.0000	9.4266	9.9524	9.9541
100.00	0.0148	0.0603	-0.0022	-0.0613	0.0000	11.8880	12.4066	12.2916

$\gamma(\%)$: Share of investment in risky assets. EC_0: Minimum single-equity contribution.
[a] Net Present Value $NPV \simeq 0$ if $|NPV| < 10^{-4}$.
[b] Recall $NPV_{EC} \simeq -NPV_P$.
[c] $DPO_i = 1$ corresponds to 1×10^{-2}. $DPO \simeq 0$ if $|DPO| < 10^{-6}$. $DPO \simeq 0$ if $|DPO| < 10^{-6}$

3. Numerical Illustration

Accounting framework (I):

Again, we can derive fair pricing – for the portfolio as a whole and for each generation – and equal default ratios in the case without any default risk

Table 6: Combination of minimum investment share in risky assets and minimum equity contribution such that the insurance portfolio is fair priced in total for the accounting framework.

	Single-equity contribution at t=0			Periodic equity contribution at issuances		
Parameter	$P_i =$	$P_i \nearrow$	$P_i \searrow$	$P_i =$	$P_i \nearrow$	$P_i \searrow$
Risky Asset proportion						
$\gamma_{min}(\%)$	9.87	9.88	9.87	9.87	9.88	9.87
Equity contributions						
EC_0	0.1663	0.2586	0.1303	0.0300	0.0300	0.0300
EC_1	0.0000	0.0000	0.0000	0.3997	0.3988	0.3795
EC_2	0.0000	0.0000	0.0000	0.4867	0.4775	0.4438
Policyholders NPVs[a]						
NPV_{1,τ_1}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
NPV_{2,τ_2}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
NPV_{3,τ_3}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Shareholders NPV[a]						
NPV_{EC}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

3. Numerical Illustration

Accounting framework (II):

Table 6: Combination of minimum investment share in risky assets and minimum equity contribution such that the insurance portfolio is fair priced in total for the accounting framework.

	Single-equity contribution at t=0			Periodic equity contribution at issuances		
Parameter	$P_i =$	$P_i \nearrow$	$P_i \searrow$	$P_i =$	$P_i \nearrow$	$P_i \searrow$
Risky Asset proportion						
$\gamma_{min}(\%)$	9.87	9.88	9.87	9.87	9.88	9.87
Shareholders NPV[a]						
NPV_{EC}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Policyholders DPO[b]						
DPO_1	0.0000	0.0000	0.0001	0.0021	0.0029	0.0019
DPO_2	0.0002	0.0002	0.0001	0.0006	0.0016	0.0003
DPO_3	0.0002	0.0003	0.0006	0.0002	0.0008	0.0000

$P_i =$: All policyholders pay the same premium 1 C.U. at issuance $P_i \nearrow$: Premiums paid by new generations increase by 0.5 C.U. $P_i \searrow$: Premiums paid by new generations decrease by 0.25 C.U.

[a] NVP: Net present value. $NPV \simeq 0$ if $|NPV| < 10^{-4}$.

[b] DPO: Default ratio. $DPO_i = 1$ corresponds to 1×10^{-2}. $DPO \simeq 0$ if $|DPO| < 10^{-6}$.

3. Numerical Illustration

Accounting framework (III):

The portfolio can be fairly priced, but NPVs for different policyholder generations and corresponding default ratio vary

Table 7: Strategies of single-equity contributions for different shares of investment in risky assets. Net present values and default ratios when all generations pay $P = 1$ C.U.

		Policyholders			Insurer	Default ratios		
$\gamma(\%)$	EC_0	NPV^a_{1,τ_1}	NPV^a_{2,τ_2}	NPV^a_{3,τ_3}	NPV^{ab}_{EC}	DPO^c_1	DPO^c_2	DPO^c_3
9.87	0.1663	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0002
10.00	0.0729	0.0001	0.0000	0.0000	0.0000	0.0144	0.0239	0.0264
15.00	0.0119	-0.0005	-0.0006	0.0017	0.0000	0.8244	0.8014	0.6007
20.00	0.0082	-0.0012	-0.0004	0.0034	0.0000	1.5228	1.4235	1.1086
25.00	0.0071	-0.0017	-0.0002	0.0050	0.0000	2.1725	2.0228	1.6154
30.00	0.0068	-0.0021	0.0000	0.0062	0.0000	2.8035	2.6153	2.1292
50.00	0.0073	-0.0034	0.0005	0.0109	0.0000	5.2027	4.8957	4.1316
80.00	0.0099	-0.0050	0.0010	0.0173	0.0000	8.5446	8.0957	6.9917
100.00	0.0123	-0.0061	0.0011	0.0211	0.0001	10.6365	10.1155	8.8189

$\gamma(\%)$: Share of investment in risky assets. EC_0: Single-equity contribution
[a] NPV: Net Present Value $NPV \simeq 0$ if $|NPV| < 10^{-4}$.
[b] NPV_{EC}: Net present value shareholders.
[c] DPO: Default ratio. $DPO_i = 1$ corresponds to 1×10^{-2}. $DPO \simeq 0$ if $|DPO| < 10^{-6}$.

3. Numerical Illustration

Accounting framework (IV):

For γ = 9.87 % we get the same results

For higher shares of risk assets, the required equity capital in the accounting model is substantially lower compared to the basic model

In the accounting model later generations are better of

Default ratios develop differently, NPV discrepancies are smaller compared to the basic model

In general, one combination of fair pricing and positive but equal default ratios exist (solvency requirements may not allow such combinations)

However, pension schemes are typically not based on the accounting model framework

3. Numerical Illustration

Accounting model (V):

Table 8: Combinations of periodic equity contributions for equal default risk for all policyholders

	Equity contribution			Policyholders			Insurer	Default ratios			
$\gamma(\%)$	EC_0	EC_1	EC_2	NPV^a_{1,τ_1}	NPV^a_{2,τ_2}	NPV^a_{3,τ_3}	NPV^a_{EC}	DPO^b_1	DPO^b_2	DPO^b_3	Δ^{max}
9.87	0.0238	0.0338	0.0189	-0.0003	-0.0003	-0.0003	0.0009	0.0279	0.0279	0.0280	0.38
10.00	0.0213	0.0314	0.0107	-0.0002	-0.0002	-0.0002	0.0007	0.0462	0.0463	0.0462	0.42
15.00	0.0294	0.0488	0.0113	0.0075	0.0076	0.0073	-0.0205	0.1859	0.1860	0.1859	0.34
20.00	0.0350	0.0618	0.0035	0.0139	0.0140	0.0132	-0.0362	0.4483	0.4489	0.4483	0.25
25.00	0.0394	0.0715	-0.0038	0.0188	0.0190	0.0174	-0.0469	0.8029	0.8044	0.8020	0.39
30.00	0.0531	0.0907	0.0250	0.0259	0.0261	0.0242	-0.0653	1.0000	1.0000	1.0003	0.35
50.00	0.1138	0.1944	0.1005	0.0661	0.0673	0.0648	-0.1735	1.0826	1.0827	1.0850	0.40
80.00	0.1250	0.2557	-0.0357	0.0794	0.0807	0.0708	-0.1843	4.0524	4.0523	4.0517	0.38
100.00	0.1613	0.3438	-0.0618	0.1016	0.1033	0.0897	-0.2341	5.2360	5.2378	5.2263	0.37

Δ_{max}: Maximum difference in percentage between policyholders default ratio.
[a] NPV: Net Present Value. $NPV \simeq 0$ if $|NPV| < 10^{-4}$.
[b] DPO: Default ratio. $DPO_i = 1$ corresponds to 1×10^{-2}. $DPO \simeq 0$ if $|DPO| < 10^{-6}$.

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4. Conclusion

In practice, we will face some wealth transfer within policyholder generations and between shareholders and policyholders

In addition, default ratios are typically different for each generation

Basic model: Fair pricing for the portfolio may be possible, but default ratios (if > 0) vary – NPVs for each generation differ too –. “Early” generations are better off

Accounting model: A combination of equity contributions and asset allocation may exist, where fair pricing is possible and equal default ratios can be provided to each policyholder generation

However, even for the accounting model, it is not possible to fix a desired or required safety level for all policyholder generations and obtain fair pricing

THANK YOU

Q&A