Advanced Conditional Risk Measurement and Risk Aggregation with Applications to Credit and Life Insurance

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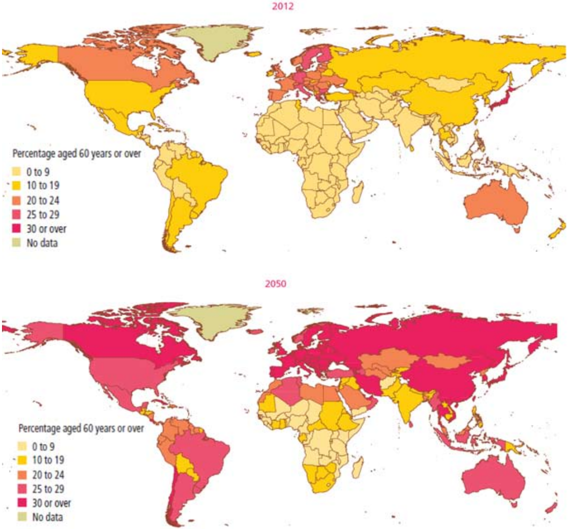
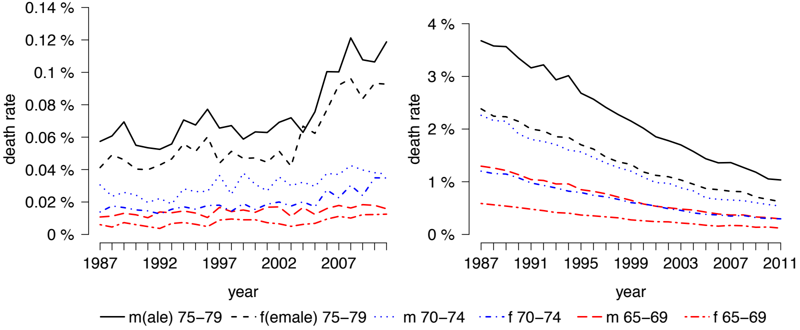
Observation I: Population aging

UN data (2012): Percentage aged 60+ years, 2012/2050 forecast



Observation II: Patterns in death cause intensities

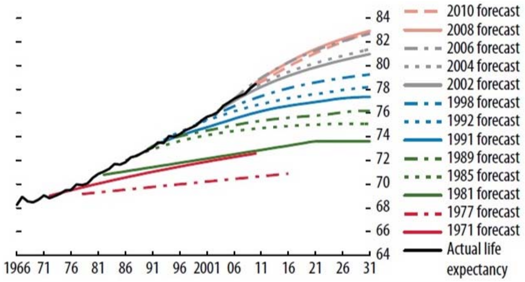
Australian **mortality rates due to different death causes** show significant patterns (1987–2011), e.g., mental and behavioural disorders (left) and circulatory diseases (right).



Observation III: Longevity risk

When applied to annuities, death probabilities should be reduced and trends should be considered to account for **longevity**. DAV:

- Mortality trends, e.g. Lee–Carter model
- $\sim 7\%$ risk margin for statistical fluctuation
- 10% risk margin for parameter risk, structural differences
- 15% risk margin for selection risk



UK, projected life expectancy at birth for males based on period life tables.

Office of National Statistics

Goal

Develop a model which derives **loss distributions** of annuity portfolios over one period which

- takes into account most risks w.r.t. **longevity**,
- accounts for changes in rates of different **death causes**,
- accounts for dependence between policyholders,
- has potentially **short execution times** (not Monte Carlo),
- can be calibrated with available data.

Collective risk model **extended CreditRisk$^+$** (see Schmock (2016), short ECRP) is able to cover all those attributes, if

default is treated as **death**.

1. Introduction
2. The model based on extended CreditRisk^{+} (ECRP)
   - Notation and setup
   - Annuity model using ECRP
3. Estimation
   - Estimation approaches
   - Risks: Back to the introduction
4. Real world example
   - Estimation results
   - Portfolio applications
5. Further applications

Notation and setup

- **Policyholders** $1, \ldots, m$.
- $\mathbb{N}_0$-valued **death indicators** $N_1, \ldots, N_m$ where $\{N_i = 0\}$ indicates 'no death'.
- $\mathbb{N}_0$-valued independent **payments** (annuity payments, premiums, discounted actuarial reserve, etc.) $X_1, \ldots, X_m$ to or from policyholders given **survival** and corresponding payments $Y_1, \ldots, Y_m$ which need not be paid in case of **death**.

Total portfolio loss

For i.i.d. copies $\{Y_{i,j}\}_{j \in \mathbb{N}}$ of Y_i, for $i \in \{1, \ldots, m\}$, derive

$$L := \sum_{i=1}^{m} X_i - \sum_{i=1}^{m} \sum_{j=1}^{N_i} Y_{i,j} \,.$$

Death indicator

Which assumptions should be satisfied by death indicators N_i in

$$S := \sum_{i=1}^{m} \sum_{j=1}^{N_i} Y_{i,j}\,?$$

- In reality, (N_i) are **Bernoulli** distributed: just Monte Carlo.
- If (N_i) are independently **Poisson**: Panjer recursion.
- If (N_i) are **compound Poisson** distributed: iterated Panjer recursion as in (extended) CreditRisk$^+$.

Multiple deaths are not a major issue

Multiple deaths of a single policyholder can occur when using (compound) Poisson distributed deaths, but:

- Since annual death probabilities are small for most ages, multiple deaths are unlikely.
- Multiple deaths are not a major issue for longevity risk modelling.
- Approximations using Poisson sums are justified by Poisson approximation, cf. Barbour, Holst and Janson (1992).
- With proper scaling, we get accurate results.

Annuity model using extended CreditRisk$^+$

Annuity model using extended CreditRisk$^+$

For all policyholders $i \in \{1, \ldots, m\}$ we assume the following:

- **One-period death probabilities** $q_1, \ldots, q_m$.
- **Risk factors** $\Lambda_1, \ldots, \Lambda_K$ are independent and have gamma distributions with mean one and variances $\sigma_1^2, \ldots, \sigma_K^2$.
- Death indicators are split up $N_i = N_{i,0} + N_{i,1} + \cdots + N_{i,K}$ due to different risk factors (**death causes**) with corresponding **weights** $w_{i,0}, \ldots, w_{i,K} \geq 0$ such that $w_{i,0} + \cdots + w_{i,K} = 1$.
- $\mathcal{L}(N_{i,0}) = \text{Poisson}(q_i\, w_{i,0})$.
- $\mathcal{L}(N_{i,k}\,|\,\Lambda_1, \ldots, \Lambda_K) \overset{\text{a.s.}}{=} \mathcal{L}(N_{i,k}\,|\,\Lambda_k) \overset{\text{a.s.}}{=} \text{Poisson}(q_i\, w_{i,k}\,\Lambda_k)$.
- Suitable (conditional) independence assumptions on $(N_{i,k})$.

Interpretation and comments on our annuity model

- Risk factors $\Lambda_1, \ldots, \Lambda_K$ represent underlying causes of death such as neoplasms, cardiovascular diseases or idiosyncratic components. Variation in risk factors represents unexpected events such as better medication, epidemics, etc.
  - For example, a low realisation of the risk factor for neoplasms Λ_k reduces the Poisson intensity in Poisson$(q_i w_{i,k} \Lambda_k)$ for all i.
- The weights $w_{i,k}$ indicate how vulnerable policyholder i is to risk factor Λ_k.
- Our model can be generalised to losses $Y_{i,k}$ depending on death cause k.
- Certain dependence structures for risk factors can be assumed, see Rudolph and Schmock (2016).

Is CreditRisk$^+$ faster than Monte Carlo? Yes it is!

Policyholders: $m = 10\,000$
Death probability: $q_1 = \cdots = q_m = 0.015$
Losses: $Y_1, \ldots, Y_m = 1$

If N_i is Bernoulli, then S is binomial with $(10\,000, 0.015)$. Similar total variations w.r.t. this binomial distribution yield the following system times in 'R':
- **Monte Carlo**: 21.6 seconds
- **(Extended) CreditRisk$^+$**: 0.01 seconds

Available data

- Historical data of annual number of deaths $n_{a,g,k}(t) \in \mathbb{N}_0$ categorised by age $a \in \{1, \ldots, A\}$, gender $g \in \{\mathrm{f}, \mathrm{m}\}$ and death cause $k \in \{0, \ldots, K\}$ for years $t \in \{1, \ldots, T\}$.
- For Australia/Austria: Long-term data for 18 age groups, both genders with 19 death causes available (ICD-9/10).
- Corresponding historical population counts $m_{a,g}(t)$.

Data and model linkage

$n_{a,g,k}(t)$ corresponds to a realisation of the random variable

$$N_{a,g,k}(t) := \sum_{i=1}^{m_{a,g}(t)} N_{i,k}(t)\,,$$

Simplifying assumptions for consistent estimation

Additionally we assume the following:

- Weights and death probabilities are equal within each age category and gender.
- **Death probabilities** in year t with year of birth $z_{a,g}$ take the form
$$q_{a,g}(t) := F^{\mathrm{Lap}}\left(\alpha_{a,g} + \beta_{a,g}\mathcal{T}_{\zeta_{a,g},\eta_{a,g}}(t) + \kappa_{z_{a,g}}\right),$$
[$F^{\mathrm{Lap}}(x) \approx \exp(-x)/2$ and $\mathcal{T}_{\zeta,\eta}(t) \approx t$] as well as **weights**
$$w_{a,g,k}(t) := \frac{\exp\left(u_{a,g,k} + v_{a,g,k}\mathcal{T}_{\phi_k,\psi_k}(t)\right)}{\sum_{j=0}^{K}\exp\left(u_{a,g,j} + v_{a,g,j}\mathcal{T}_{\phi_j,\psi_j}(t)\right)}.$$
- Risk factor variances $\sigma_1^2, \dots, \sigma_K^2$ are constant over time.
- Random variables are constant over time.

Estimation procedures: possible approaches

- **Matching of moments**: Easy to calculate and reasonably accurate.
- **Maximum likelihood (MLE)**: ML-function is given explicitly but deterministic numerical optimisation is impossible ($\sim$360 parameters in example below).
- **Maximum a posteriori (MAP)**: Similar as MLE where risk factors are not integrated out. Risk factor realisations (stress testing) and handy approximations can be derived.
- **Markov chain Monte Carlo**: Based on MLE or MAP, switching to a Bayesian setting, parameters can be estimated accurately. Samples from posterior densities are drawn.

Risks considered in our model when applied to a portfolio of life insurance contracts

- Mortality trends: Via **trends** in death probabilities & weights.
- Statistical fluctuation: Via **risk aggregation** in our model.
- Random changes in death causes: Via stochastic **risk factors**.
- Parameter risk: Via **MCMC** & sampled posterior distribution.
- Structural differences and selection risk: Possible if portfolio data available.

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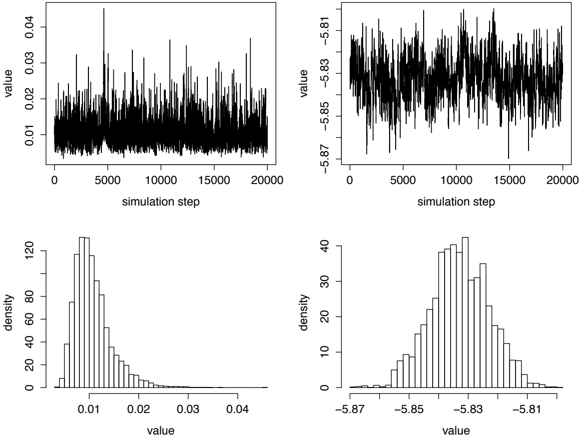
Real world example: Setup

- Australian death and population data
- **Periods** $t \in \{1987, \dots, 2011\}$.
- Eight **age categories** 50–54 years,. . . , 80–84 years and 85+ for each gender.
- Ten non-idiosyncratic **risk factors** (death causes) $\Lambda_1, \dots, \Lambda_{10}$.
- In this setting optimisation over 362 parameters is required.

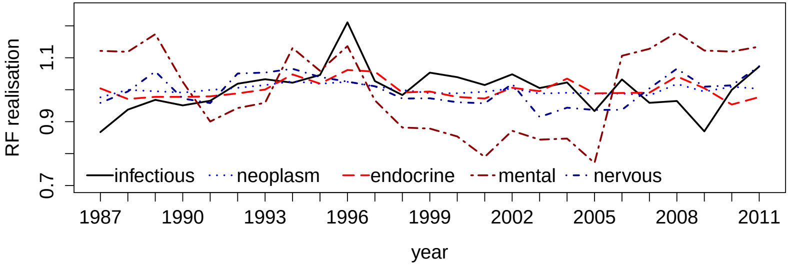
Using the extended CreditRisk$^+$ setup with trends for death probabilities and weights, we estimate the model via **matching of moments** and **MCMC** with 40 000 samples.

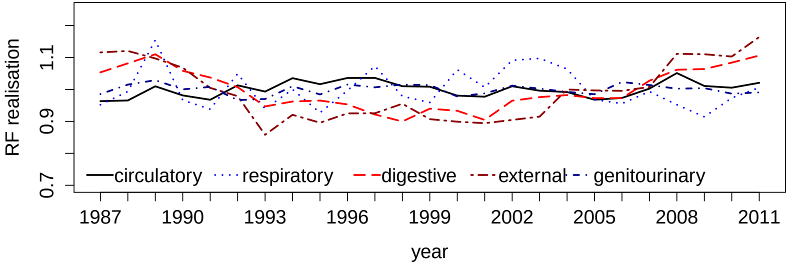
Real world example: MCMC density histograms

MCMC chains (random walk Metropolis–Hastings within Gibbs) for variance σ_1^2 of risk factor for infectious and parasitic diseases (left) as well as for parameter $\alpha_{2,\mathrm{f}}$ (right).



Real world example: Risk factor realisations





A massive increase in mental and behavioural disorders

Leading death causes for Australia and Austria.

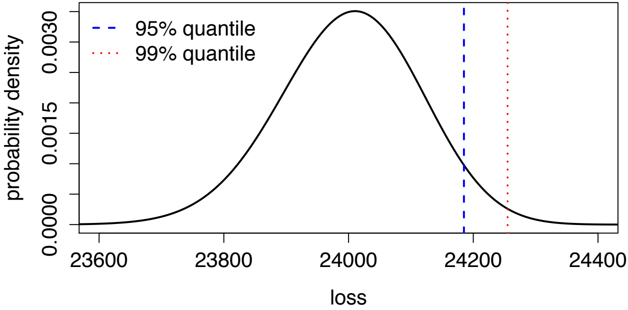
	Australia		Austria	
	2011	2041	2011	2041
	male, 85+ years			
1.	circ. (40%)	neopl. (24%)	circ. (59%)	circ. (46%)
2.	neopl. (22%)	ment. (23%)	neopl. (17%)	neopl. (10%)
3.	resp. (12%)	circ. (19%)	resp. (08%)	ment. (10%)
	female, 85+ years			
1.	circ. (44%)	ment. (34%)	circ. (67%)	circ. (51%)
2.	neopl. (13%)	circ. (17%)	neopl. (11%)	ment. (12%)
3.	ment. (10%)	neopl. (12%)	resp. (05%)	neopl. (09%)

90% quantile range: 0.01 0.03 0.05 0.07 0.09 0.11 0.13 0.15

Real world example: A simple portfolio

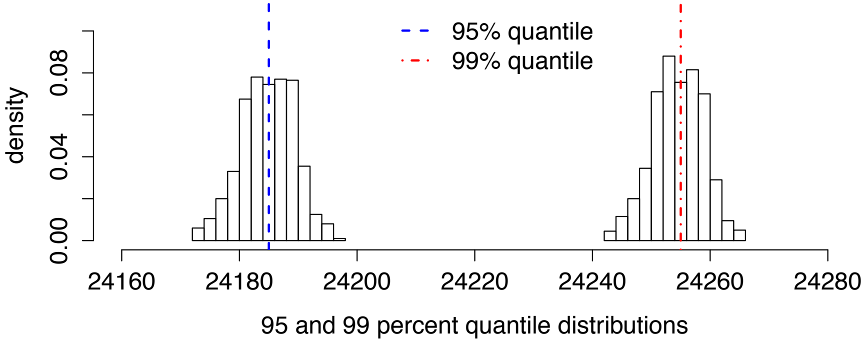
- Australian data with the same setup as before.
- Each age category and gender contains 10 policyholders with annual annuities $11, \dots, 20$.

Derive **loss distribution** $L = \sum_{i=1}^{m} X_i - \sum_{i=1}^{m} \sum_{j=1}^{N_i(T+1)} X_{i,j}$ with extended CreditRisk$^+$ where $X_{i,j} \sim X_i$.



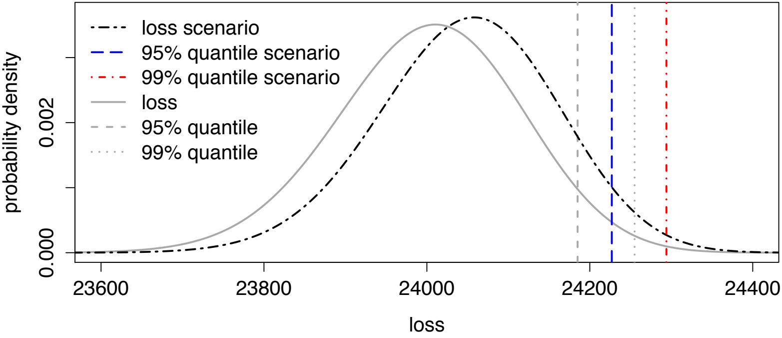
Real world example: Parameter uncertainty

MCMC yields approximations for **distributions of quantiles** of L, i.e., we can quantify parameter risk.



Real world example: Scenario analysis

If **deaths due to neoplasms decrease by 25%** over all ages, we can estimate risk factor realisation $\lambda_{\text{neo}} = 0.7991$ and get the distribution of L under this scenario.



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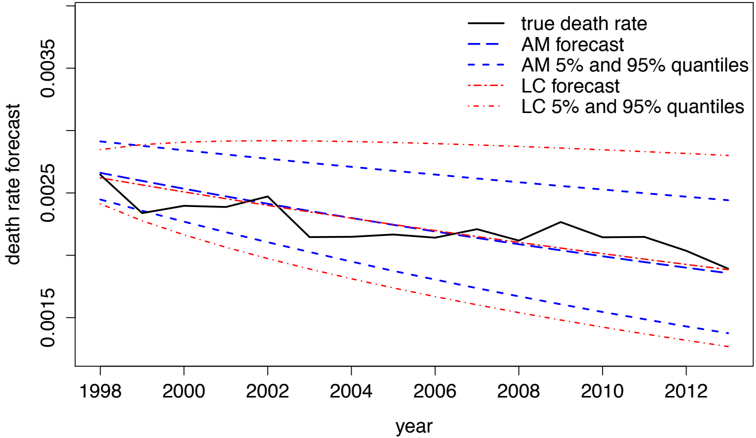
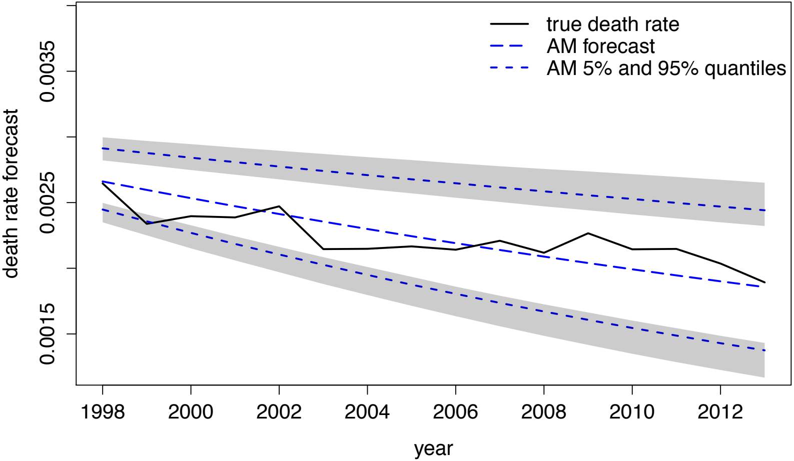
Forecasting death rates with our model vs. Lee–Carter

Using our annuity model, we can **forecast death rates** via setting $Y_i = 1$. Comparing estimated bounds with bounds obtained by the Lee–Carter model for females aged 55 to 59 gives:



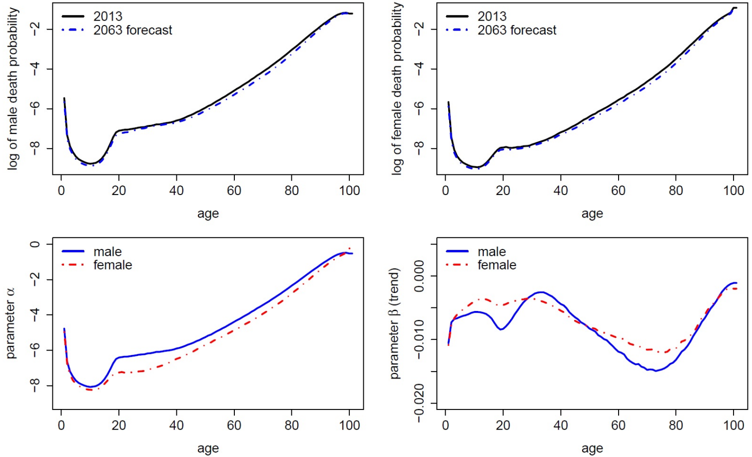
Forecasting death rates and parameter uncertainty

Parameter uncertainty is given by shaded areas (90 percent confidence bands) of 5%- and 95% quantiles:

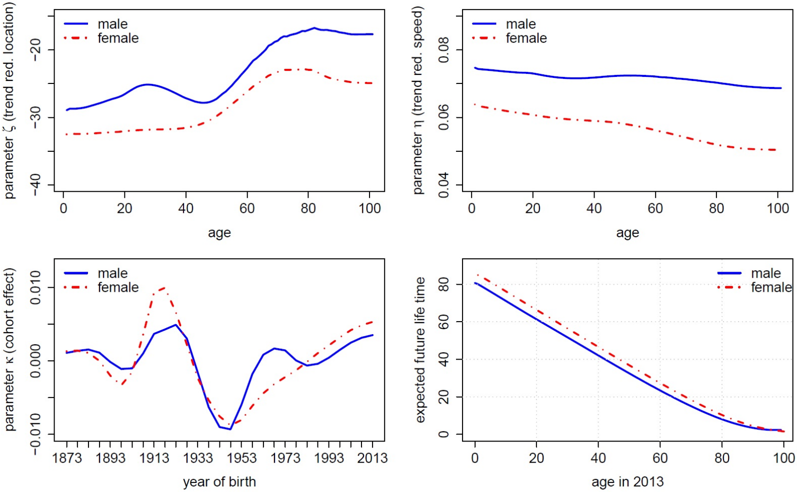


Life tables via Markov chain Monte Carlo: Australia I

We use Australian death data for the period $\{1971, \ldots, 2013\}$.
Our model with just idiosyncratic risk is able to **forecast death probabilities** via $q_{a,g}(t) = F^{\mathrm{Lap}}\left(\alpha_{a,g} + \beta_{a,g}\,\mathcal{T}_{\zeta_{a,g},\eta_{a,g}}(t) + \kappa_{z_{a,g}}\right)$.



Life tables via Markov chain Monte Carlo: Australia II



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