Safe Now or Sorry Tomorrow - The Impact of Time-Structure on Optimal Loss Prevention

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Motivation

- Although the effect of risk preferences on prevention decisions has long been studied by economists, the overall focus is on *static* models.
- This is surprising given the fact that many prevention decisions can be viewed as an agent's investment in future states.
- Building on recent literature, we characterize optimal prevention decisions in a broader intertemporal context by analyzing both investment in prevention today and tomorrow in a *two-period* modeling approach.

Some Practical Examples

- stop smoking today $\rightarrow$ positive effect of today and future lifetime health
- vaccination against some infectious disease today $\rightarrow$ lower/no infection risk today and in the future
- safety driving course $\rightarrow$ better driving today and potentially less accidents in the future

"in these cases the usual one-period framework cannot be used to analyze the optimal level of prevention. A two-period framework is suitable." (Menegatti (2009), p. 394)

Outline

- We extend the classical Ehrlich and Becker (1972) self-protection model by adding prevention today with preventive effect tomorrow.
- We analyze demand for such generalized prevention decisions from an ex-ante perspective.
- We discuss comparative statics results for both a risk-neutral and a risk-averse decision maker.
- Finally, we introduce market insurance and analyze the relationship between insurance and generalized self-protection measures.

Related Literature

- Dionne and Eeckhoudt (1985): More risk-averse agents invest more in self-insurance.
- Dionne and Eeckhoudt (1985): Increasing risk aversion has an ambiguous effect on the optimal level of prevention.
- Eeckhoudt and Gollier (2005): Under specific assumptions on optimal prevention by a risk-neutral agent, a prudent (imprudent) agent selects a level of effort that is smaller (greater) than that of the risk-neutral decision-maker.
- Menegatti (2009): Shows in a two-period prevention model that prudent (imprudent) agents spend *more* (less) in prevention than risk-neutral ones. Here, spending on effort and the resulting effects of this effort take place at different points in time.

$\rightarrow$ assumptions on the timing of prevention are crucial!

Model

- Consider a consumption pattern (c_1, c_2) associated with utility $V = u(c_1) + \delta u(c_2)$, where $0 < \delta \leq 1$ is the discount rate.
- *Anticipatory prevention* or prevention now is conducted, if the agent spends e_1 dollars in t_1 (or "today") to benefit from a decrease in loss probability *in the following period* t_2 (or "tomorrow").
- *Contemporaneous prevention* or prevention then is the case, if the agent decides to spend e_2 dollars in t_2 to benefit from a reduced loss probability *in the same period* t_2.
- Marginal prevention cost is normalized to 1.
- Overall expected utility is

$$V(e_1, e_2) = u(w_1 - e_1) + \delta \cdot [p(e_1, e_2) u(w_2 - e_2 - L) + (1 - p(e_1, e_2)) u(w_2 - e_2)].$$

- We assume $p_i(e_1, e_2) < 0$, but at a decreasing rate so that $p_{ii}(e_1, e_2) > 0$ for $i \in \{1, 2\}$.

Optimal prevention for a risk-neutral agent

For a risk-neutral agent, the optimal prevention problem has at least one solution. In case of an interior solution, discounted marginal loss prevention on anticipatory prevention equals marginal prevention of contemporaneous prevention.

- Intuition: Problems share that marginal benefit for each component of prevention technology has to coincide with its marginal cost at the optimum. However, in the combined model, we additionally observe that optimality requires a certain balance between the two components. Therefore, the ratio of discounted productivity on prevention now by prevention then equals the ratio of marginal cost now and then, which is given by $(-1)/(-1) = 1$.

If there is an interior solution, overall convexity of prevention technology is sufficient for maximality.

Optimal prevention for a risk-neutral agent

- For higher values of δ, and therefore with weaker discounting, investment in e_1 will increase, whereas the relation regarding e_2 depends on the sign of p_{12}: $p_{12} < 0$ entails a positive relationship for e_2 and $p_{12} > 0$ a negative one.
- Given $p_{12} < 0$, i.e., if prevention is submodular, a higher loss severity entails both more investment in prevention today and more investment in prevention tomorrow.
- Prevention now and then are complements if and only if $p_{12} < 0$.

- Intuition: $p_{12} < 0$ means that with higher spending on anticipatory prevention, contemporaneous prevention becomes more efficient!
- A negative cross derivative is referred to as submodularity (Milgrom and Roberts (1990)) in the economics literature.

Numerical Example

We use $w_1 = 100, w_2 = 150, L = 50, \delta = 0.98$.
Prevention technology is assumed to be $p(e_1, e_2) = f_1(e_1) + f_2(e_2)$ with $f_i(e) \equiv \exp\left(\frac{\alpha_i}{e-100} + \beta_i\right), \alpha_1 = 900, \alpha_2 = 700$ and $\beta_i = -\log(2) + \frac{\alpha_i}{100}$.

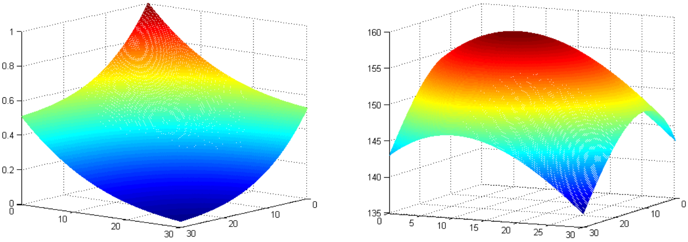

Figure: Overall convexity of prevention (left) and concave overall utility (right) for a risk-neutral decision maker.

Optimal prevention with this specific parametrization can be identified as $(e_1^*, e_2^*) = (10.02, 9.88)$.

Optimal prevention for a risk-averse agent

A risk-averse agent is more productive on (discounted) anticipatory prevention than on contemporaneous prevention if and only if marginal utility today exceeds expected marginal utility tomorrow.

- Intuition: Optimal prevention balances the two prevention components in such a way that marginal utility today and tomorrow form the same proportion as discounted efficiency of prevention today and tomorrow:

$$\delta \frac{p_1}{p_2} \frac{[u_2 - u_1]}{[u_2 - u_1]} = \frac{u_0'}{p u_2' + (1-p) u_1'},$$

- Hence, the ratio of discounted marginal benefit on anticipatory prevention by marginal benefit of contemporary prevention has to be equal to the ratio of respective marginal costs measured in utility terms.

Optimal prevention for a risk-averse agent

- Wealth today is positively related to prevention today, and also to prevention tomorrow if in addition $p_{12} < 0$.
- The effect of an increase in future wealth is ambiguous.
- Weaker discounting enhances prevention today, and also prevention tomorrow if in addition $p_{12} < 0$.
- Under $p_{12} < 0$ and moderate risk aversion, both prevention today and tomorrow increase in the loss size. If prevention is sufficiently supermodular and risk aversion sufficiently high, prevention today increases whereas prevention tomorrow decreases.
- Most importantly, prevention today and tomorrow can be substitutes or complements. Sufficient for complementarity is $p_{12} < 0$.

- Intuition: Higher wealth today reduces relative cost of prevention today. $p_{12} < 0$: increased prevention today makes prevention tomorrow more efficient $\rightarrow$ higher levels of e_2.

Market Insurance

- Assuming the premium is due today, i.e., in period t_1, the individual's expected utility over both periods is

$$Z = u(w_1 - e_1 - \alpha\pi L) + \delta \cdot [p(\cdot) u(w_2 - e_2 - (1-\alpha)L) + (1 - p(\cdot)) \cdot u(w_2 - e_2)],$$

  where α is the proportion of insurance, π is the price of insurance, $\alpha\pi L$ represents the insurance premium, and $p(\cdot)$ is the probability of loss.

- We analyze three cases:
  1. full information scenario
  2. only period-1 prevention observable (and contractible) by the insurer
  3. both prevention now and then unobservable by the insurer

Market Insurance - full information case

- Now the insurer can use both prevention at t_1 and t_2 as pricing information (no moral hazard)!
- The policyholder is able to reduce the insurance premium by both contemporaneous and anticipatory prevention.
- The premium rate is given by $\pi = p(e_1, e_2) + \lambda$.

- If prevention is submodular, raising the loading factor has an ambiguous effect on the extent of optimal prevention. If prevention is sufficiently supermodular, prevention today will decrease and prevention tomorrow will increase after an increase in the loading.
- More insurance has an ambiguous effect on prevention now and then.
- If discounted marginal utility in the loss state and marginal utility now do not differ too much and prevention is sufficiently supermodular, higher levels of insurance will induce a decrease in prevention now and an increase in prevention then.

Market Insurance - period-1 prevention observable

- Now we assume $\pi = p(e_1, 0) + \lambda$ so that the price of insurance depends on the level of prevention in period 1 of the policyholder.

- If insurance is loaded more heavily, prevention now will be reduced and so will be prevention then under submodularity.
- In general, more insurance has an ambiguous effect on prevention now and then.
- Finally, if prevention is submodular and absolute risk aversion is sufficiently low, there is substitution between insurance and prevention.

- Intuition: Higher values of $\pi \rightarrow$ less wealth today $\rightarrow$ less prevention today! $p_{12} < 0$: decrease in prevention today makes prevention tomorrow less effective!

Market Insurance - prevention unobservable by insurer

- We assume $\pi = p_0 + \lambda$ with $p_0 \equiv p(0,0) > p(e_1, e_2) \; \forall (e_1, e_2) \neq 0$.
- Note that in case $\lambda = 0$ the insurance premium $\alpha \pi L$ is actuarially fair for no prevention.

- A higher price of insurance unambiguously decreases optimal prevention today. It decreases also optimal prevention tomorrow if $p_{12} < 0$.
- In general, for higher values of α, and therefore with higher coinsurance, investment in e_1 and e_2 will increase or decrease.
- If prevention is submodular and if absolute risk aversion is sufficiently low, then insurance and prevention are substitutes.

- Note: By comparing the last two scenarios, we see that qualitatively they do not differ. Hence, the moral hazard issue along e_2 is so strong as to undermine the additional benefit of observability of e_1!

Summary

- Given an intuitive assumption on the nature of intertemporal prevention, comparative static results show that
  1. prevention now and then are normal goods with respect to period-1 wealth, but not necessarily with respect to period-2 wealth,
  2. a weaker presence bias enhances prevention,
  3. with moderate risk aversion more severe losses enhance prevention, and
  4. prevention today and in the future are complements.

- Introducing insurance into our two-period setting, our analysis confirms the classic static result from Ehrlich and Becker (1972) that insurance and loss prevention can be either substitutes or complements.
- Our findings extend this classical result by showing that under intuitive assumptions on the nature of intertemporal prevention and the level of risk aversion, insurance and prevention are substitutes.

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