Valuation and Risk Assessment of Participating Life Insurance in the Presence of Credit Risk

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Introduction

Motivation

- Credit risk has long not been in the focus of most insurance companies
- Due to the long-term and guaranteed liabilities, life insurers invest a large part of their capital in long-term assets such as corporate and government bonds
- As a consequence of financial and sovereign crises, credit risk became increasingly important in the recent years
- In participating life insurance, capital investment management decisions can substantially affect
  - Policyholders' surplus participation
  - Insurers' risk situation

Introduction

Aim of paper

- Analyze the impact of market and credit risks in participating life insurance from the equityholders' perspective
- Focus on the asset side and the induced market risk for
  - Stocks
  - Bonds (corporates and governments)
- Insurers' assets are simultaneously affected
  - Equity risk
  - Interest rate risk
  - Credit risk
- Compare the differences for fair contract parameters and risk measures when taking credit into account or not

Model framework

Company overview

- At $t = 0$
  - Policyholders' upfront premium: $P(0) = k \cdot A(0)$
  - Initial equity capital: $E(0) = A(0) - P(0)$
- Dividend payment (see Bohnert, Gatzert, and Jørgensen, 2012)

$$D(t) = \mathbb{I}_{\{A(t)-P(t)>\beta\cdot E(0)\}} \cdot \beta \cdot E(0)$$

Modeling the liabilities (see, e.g., Gatzert, 2008)

- Cliquet-style guarantee

$$P(t) = P(t-1)\cdot\left(1+r_P(t)\right) = P(t-1)\cdot\left(1+\max\left(r_g, \gamma\cdot\left(\frac{A(t)}{A(t-1)}-1\right)\right)\right)$$

- Terminal surplus participation

$$B(T) = \max\left(A(T)-P(T), 0\right)$$

Model framework

Asset dynamics (1)

Equity risk

- Stocks follow a geometric Brownian motion

$$dA_S(t) = \mu_S \cdot A_S(t)\,dt + \sigma_S \cdot A_S(t)\,dW_S^{\mathrm{P}}(t)$$

Interest rate risk

- Non-defaultable zero bond price is defined by the Cox, Ingersoll, and Ross (1985) (CIR) model
  - Short rate *r*(*t*): $dr(t) = \kappa \cdot \left(\theta - r(t)\right)dt + \sigma_r \cdot \sqrt{r(t)}\,dW_r^{\mathrm{Q}}(t)$
  - Closed affine form for non-defaultable zero bond price:

$$p(t,h) = E_t^{\mathrm{Q}}\left(e^{-\int_t^h r(s)ds}\right) = e^{F(t,h) - G(t,h)\cdot r(t)}$$

Model framework

Asset dynamics (2)

Credit risk

- Defaultable zero coupon bond price is based on the reduced-form credit risk model by Jarrow, Lando, and Turnbull (1997) (JLT)
  - Time-homogenous Markov process: $X = \left(x(t), t \in \mathbb{N}_0\right)$
  - Distribution: $\Psi(t,h) = \begin{pmatrix} \psi_{1,1}(t,h) & \cdots & \cdots & \psi_{1,m}(t,h) \\ \vdots & & & \vdots \\ \psi_{m-1,1}(t,h) & \cdots & \cdots & \psi_{m-1,m}(t,h) \\ 0 & 0 & \cdots & 1 \end{pmatrix}$
  - Stopping time: $\tau^B = \inf\left\{t \in \mathbb{N} : x(t) = m\right\}$ and recovery rate of face value: δ

- Defaultable zero coupon bond price with rating $x(t) = i$

$$\hat{p}(t,h) = E_t^{\mathrm{Q}}\left(\mathbb{I}_{\left\{\tau^B > h\right\}} \cdot e^{-\int_t^h r(s)ds} + \mathbb{I}_{\left\{\tau^B \le h\right\}} \cdot \delta_R \cdot e^{-\int_t^h r(s)ds}\right)$$

$$= p(t,h) \cdot \left(\delta_R + \left(1-\delta_R\right) \cdot \left(1-\psi(t,h)\right)\right)$$

Model framework

Asset dynamics (3)

Market value at time t

- Stock portfolio comprising N_S exposures

$$A_S(t) = \sum_{i=1}^{N_S} A_{S,i}(t)$$

- Bond portfolio comprising N_B exposures

$$A_B(t) = \sum_{j=1}^{N_B} A_{B,j}(t)$$

with $$A_{B,j}(t) = \mathbb{I}_{\left\{\tau_j^B > t\right\}} \cdot \sum_{h=t+1}^{T_j} \left(CF_j(h) \cdot \hat{p}_{x_j(t)=i}(t,h) \right) + \mathbb{I}_{\left\{\tau_j^B = t\right\}} \cdot \delta_R \cdot A_{B,j}(t-1)$$

Model framework

Asset dynamics (4)

Management decisions regarding the asset allocation

- Market value of assets (stocks and bonds) at time *t* before (-) and after (+) rebalancing the assets (based on Gerstner et al., 2008)
  - Market value of assets at time *t*:
    $$A(t) = A_S^-(t) + A_B^-(t) + CF(t) - D(t-1)$$
  - Free capital at time *t*:
    $$F(t) = A(t) - A_B^-(t) - D(t-1) = A_S^-(t) + CF(t) - D(t-1)$$
  - Market value of stocks at time *t* after rebalancing:
    $$A_S^+(t) = \max\left(\min\left(\alpha \cdot A(t), F(t)\right), 0\right)$$
  - Market value of bonds at time *t* after rebalancing:
    $$A_B^+(t) = A_B^-(t) + F(t) - A_S^+(t)$$

Model framework

Fair valuation from the equityholders' perspective and risk measurement

- Time of company default

$$\tau^C = \inf\left\{t \in \{1,\dots,T\} : A(t) < P(t)\right\}$$

- Equity capital at time *t*

$$E(t) = \begin{cases} A(t) - L(t) & ,\tau^C > t \\ 0 & ,else \end{cases} \quad \text{with} \quad L(t) = \begin{cases} P(t) & ,(t < T_j) \wedge (\tau^C > t) \\ P(t) + \delta_L \cdot B(t) & ,(t = T_j) \wedge (\tau^C > t) \\ A(t) \cdot (1-c) & ,\tau^C = t \\ 0 & ,else \end{cases}$$

- Equityholders's claim at time *t* and fair condition

$$\Pi^E = E^{\mathrm{Q}}\left(\mathbb{I}_{\{\tau^C > T\}} \cdot \left(e^{-\int_0^T r(s)ds} \cdot E(T) + \sum_{t=1}^{T} e^{-\int_0^t r(s)ds} \cdot D(t-1) \right) + \mathbb{I}_{\{\tau^C \le T\}} \cdot \sum_{t=1}^{\tau^C - 1} e^{-\int_0^t r(s)ds} \cdot D(t-1) \right) \overset{!}{=} E(0)$$

- Net present value and shortfall probability

$$NPV^P = E^{\mathrm{Q}}\left(\mathbb{I}_{\{\tau^C > T\}} \cdot e^{-\int_0^T r(t)dt} \cdot L(T) + \mathbb{I}_{\{\tau^C \le T\}} \cdot e^{-\int_0^{\tau^C} r(t)dt} \cdot L(\tau^C) \right) - P(0) \quad \text{and} \quad SP = LPM_0 = P(\tau^C \le T)$$

Numerical results

Input parameters

- Company and contract parameters
  - $T = 15$
  - $P(0) = 85$, $E(0) = 15$, $A(0) = 100$
  - $\delta_L = 0.3$
  - $\beta = 0.06$

- Stock portfolio

i		m_S	σ_S
1	DAX 30	0.0637	0.2164
2	FTSE 100	0.0436	0.1658
3	Dow Jones Industrial	0.0755	0.1784

- CIR model

$r(0)$	0.0369	σ_r	0.0342
θ	0.0369	κ	0.1810

- Correlations (S_i, r)

- Bond portfolio

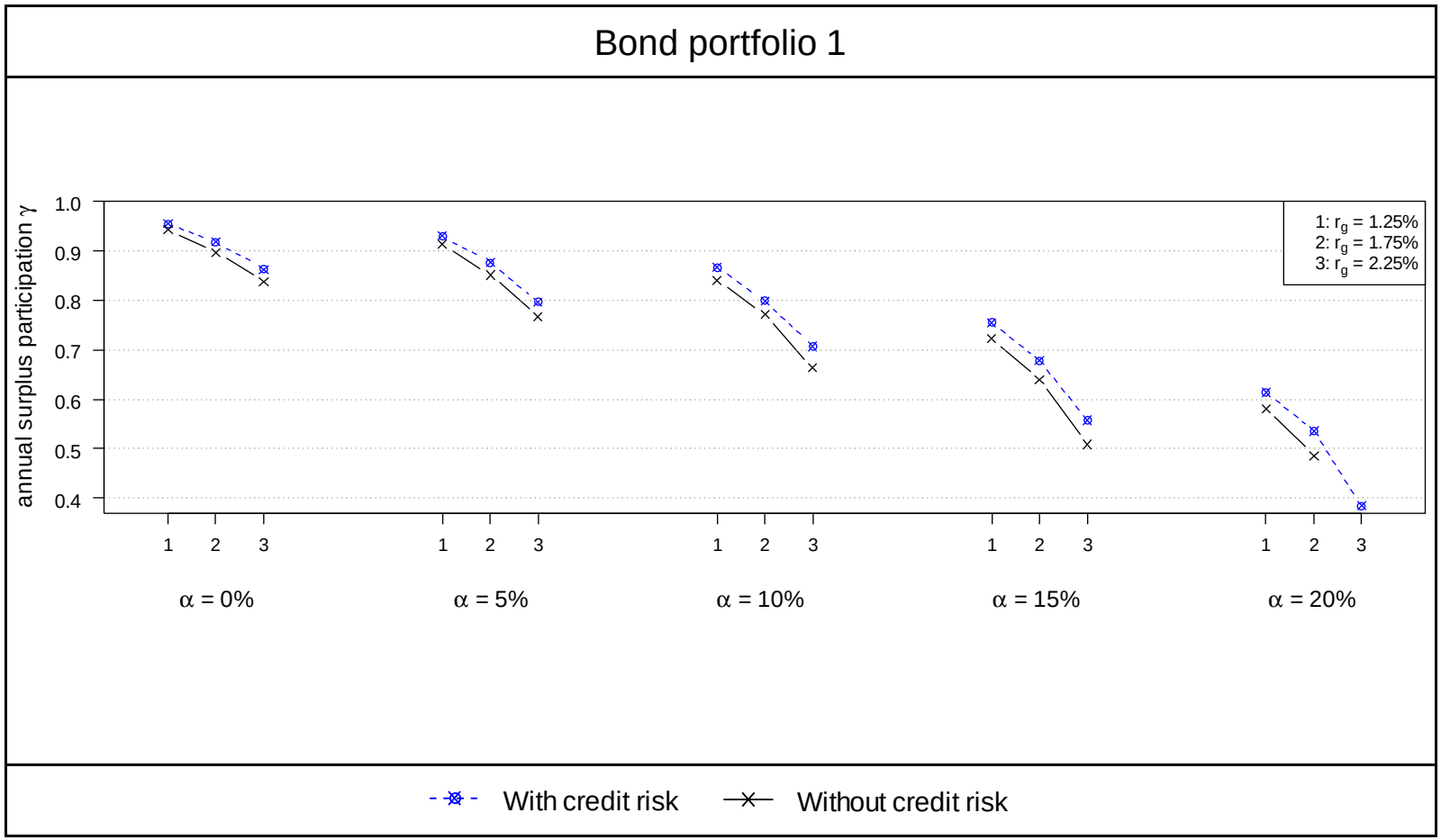
j	Type	Rating	Maturity (years)	Coupon p.a. (%)
1	Corporate	AA	15	2.800
2	Corporate	A	15	4.125
3	Corporate	BBB	15	7.125
4	Government	AAA	16	2.275
5	Government	A	15	3.000
6	Government	BB	17	7.750

- JLT model

δ_R	0.55
ψ	Transition rates from rating agencies

Numerical results

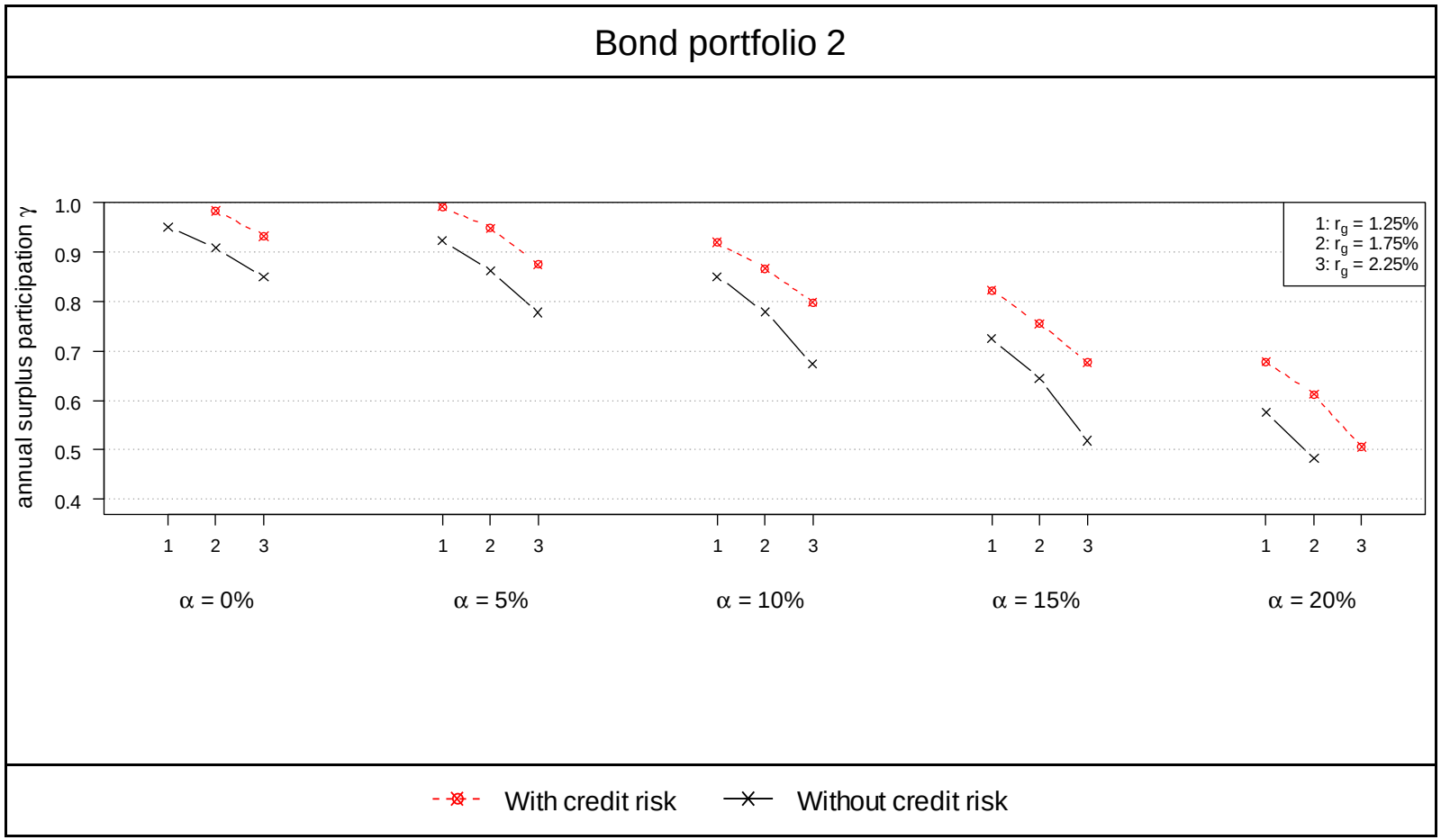
Fair contracts with and without credit risk



Notes: Bond portfolio 1 (higher grade): B_1, B_2, B_4, B_5.

Numerical results

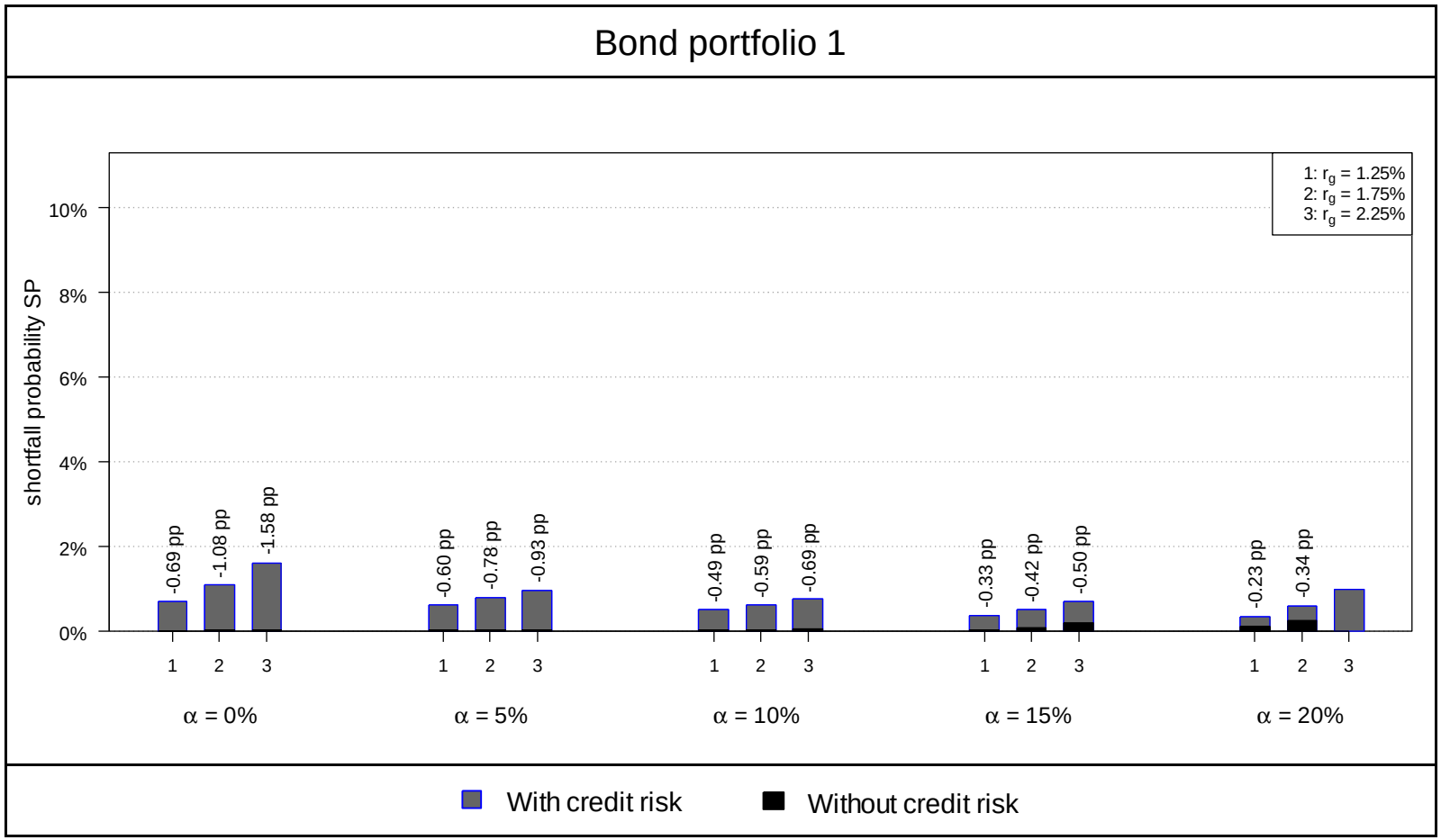
Fair contracts with and without credit risk



Notes: Bond portfolio 2 (lower grade): B_2, B_3, B_5, B_6.

Numerical results

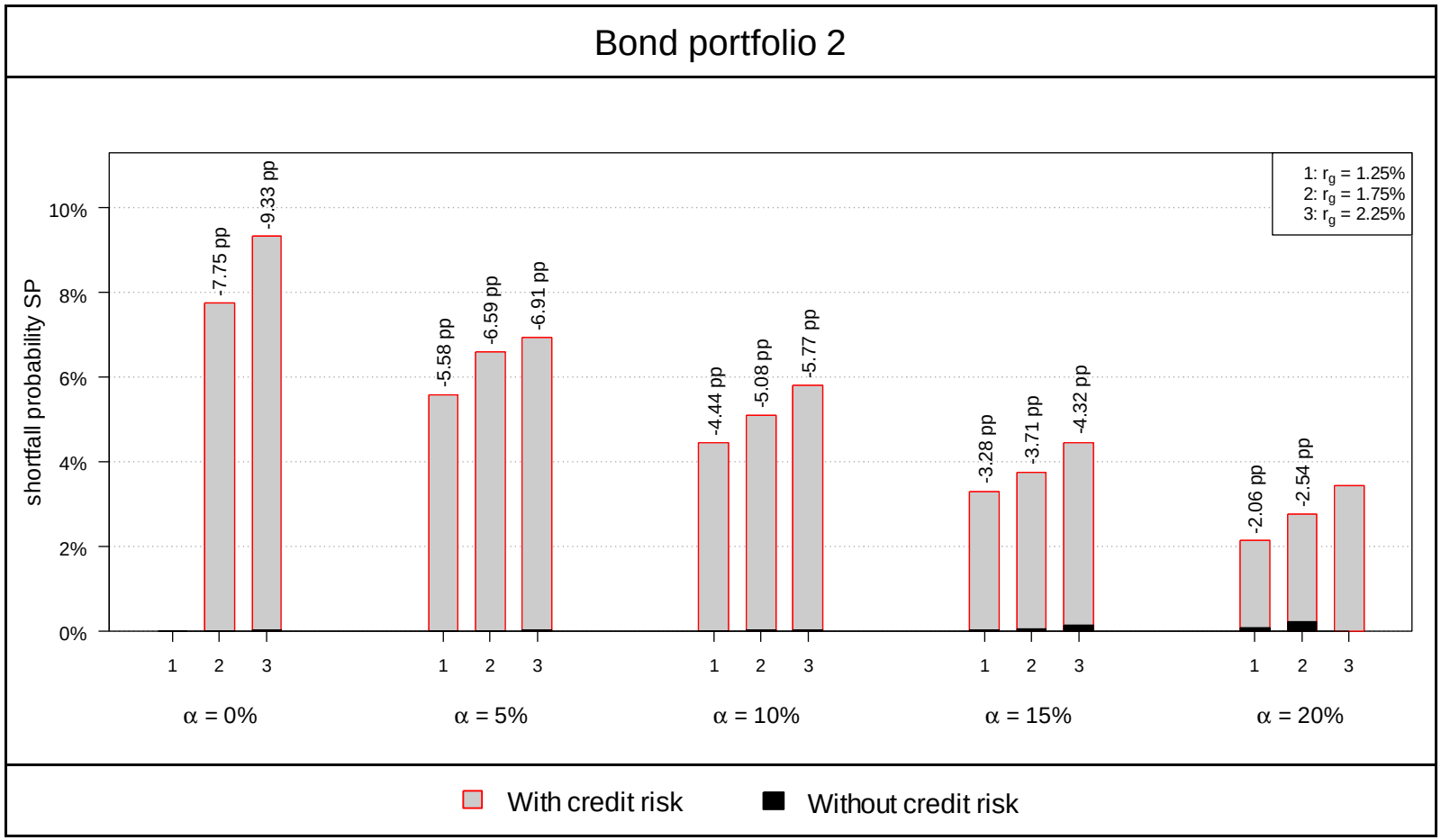
Risk measurement with and without credit risk (1)



Notes: Bond portfolio 1 (higher grade): B_1, B_2, B_4, B_5.

Numerical results

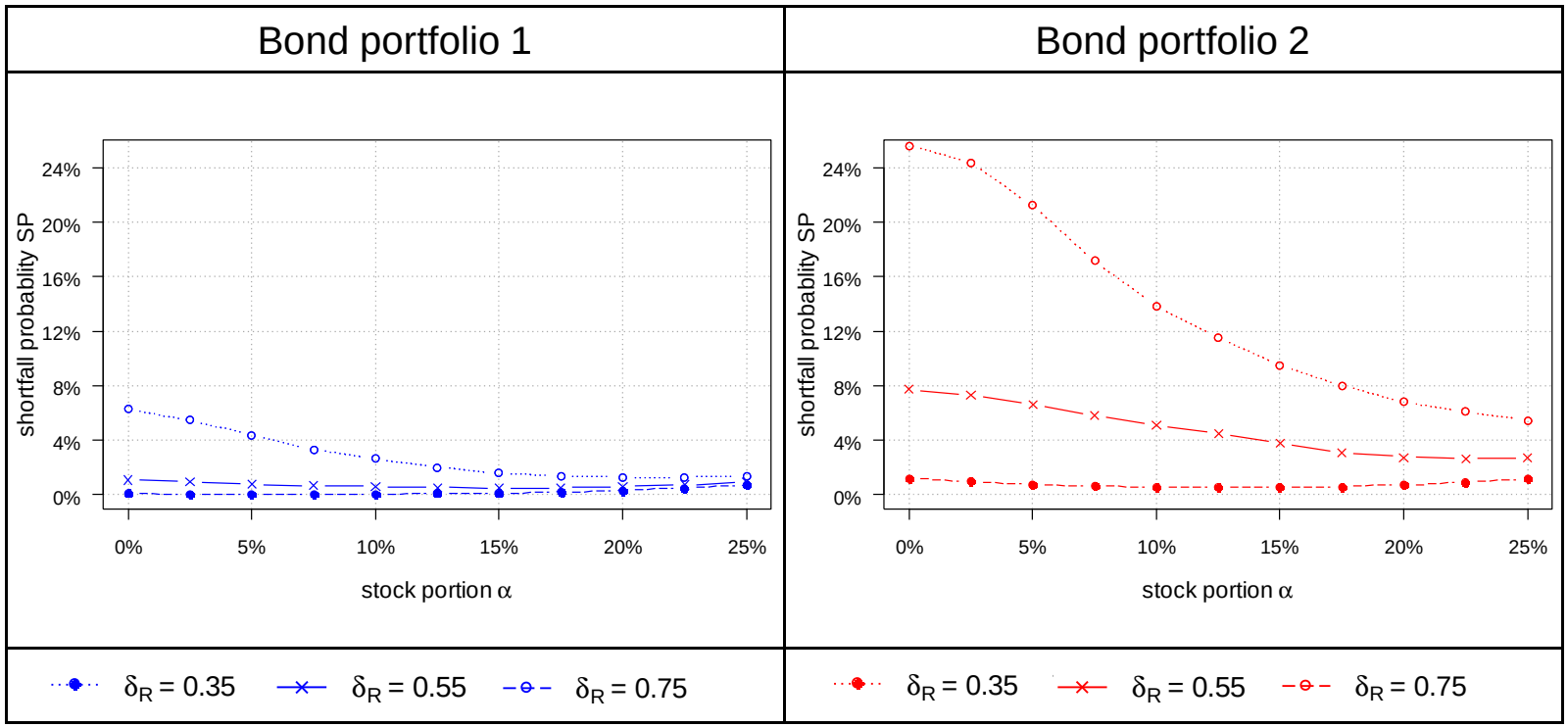
Risk measurement with and without credit risk (2)



Notes: Bond portfolio 2 (lower grade): B_2, B_3, B_5, B_6.

Numerical results

Impact of bonds‘ recovery (with r_g = 1.75%)



Notes: Bond portfolio 1 (higher grade): B_1, B_2, B_4, B_5; bond portfolio 2 (lower grade): B_2, B_3, B_5, B_6.

Summary

- Focus on participating life insurance contracts: Study the interaction between equity risk, interest rate risk, credit risk

- Impact of credit risk on the insurers' shortfall risk strongly depends on the insurer's asset allocation
  - Tradeoff between higher coupon payments (→ offered by lower grade assets to compensate investors for taking higher risk) and increase in shortfall risk: a "turning point" may exist
  - Tradeoff between equity risk and credit risk
  - ➢ Shortfall risk can even decrease for an increasing stock portion in case of high credit risk exposure in (low-grade) bond portfolios

➢ Consideration of interaction effects between different risks imposed by asset portfolio is vital for an adequate risk assessment

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Thank you very much for your attention!

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